Week 4

Topics
- *Capital structure*
- *Modigliani-Miller theorems*
- *Leverage effect*
- *Dividend policy*

Skills/calculative processes
- **Calculating the cost of equity using the DDM model formula**
- **Merton Model graph**
- **Calculating WACC**
- **Calculating returns with different capital structures and states of the world**
- **Mathematical implications of MM**
- **MM with frictions**
- **Dividend policy irrelevance calculations**

Investment decisions
- Discount rate: opportunity cost of capital required by investors given the best available expected return offered on investments of comparable risk and timing
- Flat discount rate implied in Gordon formula a simplifying assumption – usually varied
- Prices in bond market reveal time value of money, prices in stock market reveal risk premium
- In general **debt risk < firm risk < equity risk**

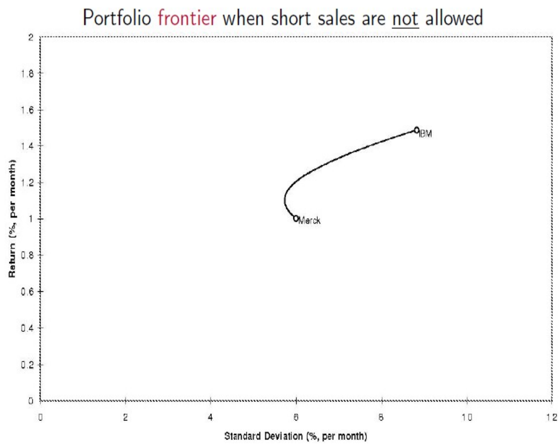
Capital structure

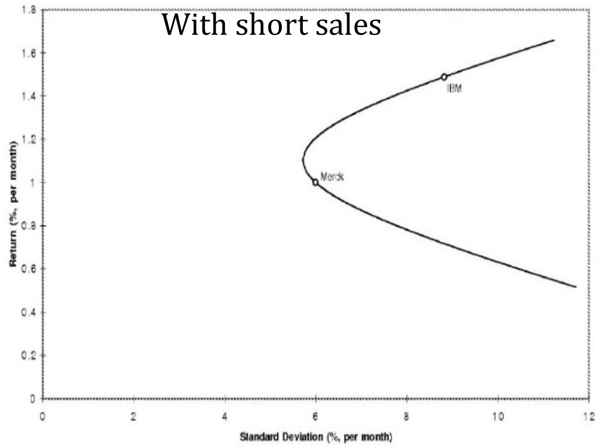
Cost of equity
- Shareholders require opportunity cost of capital (return on next best alternative of same risk)
- Dividends and capital appreciation are both sources of value
- Managers **estimate cost of equity by solving for an observed stock price an equity valuation model like the DDM** with constant growth
- **$P0 = D1/r\text{-}g$**
- In equilibrium (no arbitrage opportunities), stock price reveals correct op cost of the firm's equity

Equity vs Debt
- **Debt** – fixed claim, debt is senior to equity (repaid first), if there is no default risk, cost of debt equals return on riskless securities
- **Equity** – residual claim (cash flows subject to availability), cost of equity equals return on comparable investments
- Distinction between debt and equity only relevant if future cash flows from business uncertain – otherwise both types of claimants could forecast cash flows accurately and would require the risk free rate
- Capital structure is composition of debt/equity – the mix of equity and debt will affect the discount rate with which we discount future cash flows

Merton model
- **Assume all firm debt is a zero bond with a face value d and maturing in T**
- Graph reflects payoffs to debt – and equity holders in T as a function of firm value V
- **Upside potential to the debt is capped at d while it is unlimited for the stock**

- Short sales allow a potential for **greater returns but with higher risk**





- **Correlation period must be specified** as these change over time

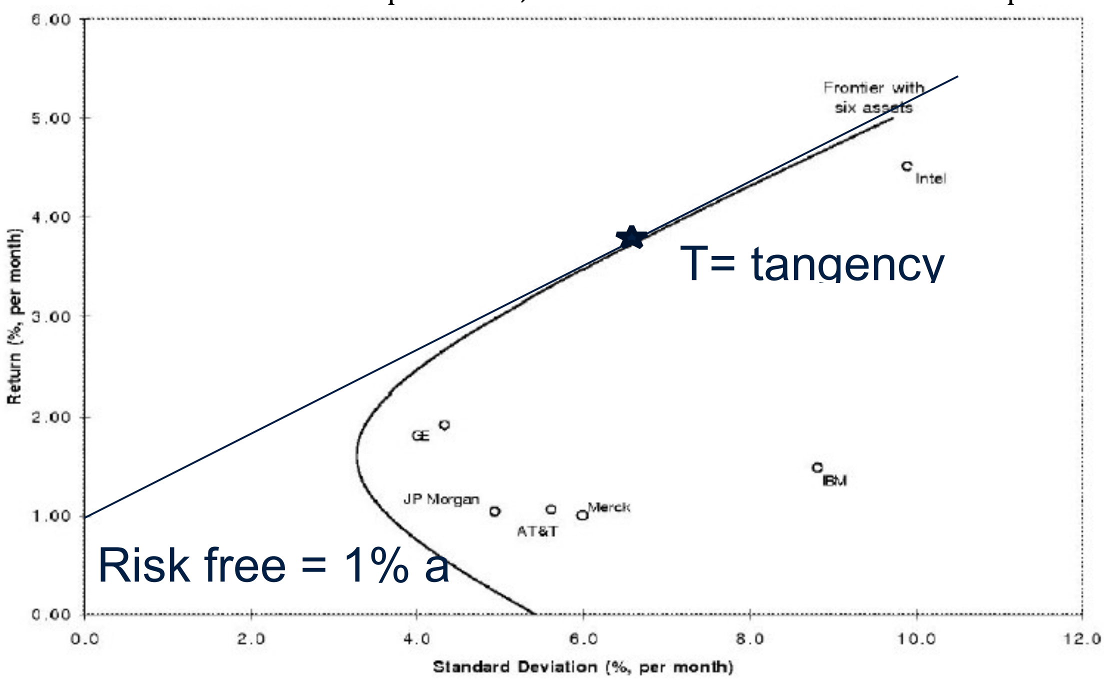
- The formula for finding the optimal portfolio weight with 2 assets is below:

$$w_A = \frac{[E(r_A) - r_f] \times \sigma_B^2 - [E(r_B) - r_f] \times \sigma_{AB}}{[E(r_A) - r_f] \times \sigma_B^2 + [E(r_B) - r_f] \times \sigma_A^2 - [E(r_A) + E(r_B) - 2r_f] \times \sigma_{AB}}.$$

- <u>Other points</u>
- When there is a portfolio of two assets with different correlation coefficients there are no benefits from diversification
- Assets won't lie on the portfolio frontier – it is through diversification between them that these risk/return combinations can be achieved
- More assets is better – frontier moves to the left – **cannot be worse as if they made the risk/return mix worse they would not be included**

Optimal portfolio selection

- **Given an expected return, the portfolio that minimizes risk is a mean-variance frontier portfolio**
- The locus of all frontier portfolios in the mean-std plane is called the portfolio frontier
- The **upper part** of the portfolio frontier gives the **efficient frontier portfolios**
- To obtain the efficient portfolios, we need to solve the constrained optimization problem



Introducing risk free rate - (Brealey, Myers and Allen, 2008)

- Suppose you can lend/borrow at rate rf (could signify treasury bills etc)
- **Draw a line from 0 standard deviation, risk free rate to being tangential with the efficient frontier**
- At the point of tangency between the line from rf and the efficient frontier, S, the **optimal portfolio** is discovered
- Then the investor is left to decide what proportion they invest in the risk free asset and what proportion in the stock portfolio, depending on their preferences for risk and return
- Any point along the line of tangency is feasible
- Can **extend beyond the point of tangency through borrowing/short sales**
- Lower the rate of risk free → lower borrowing cost → higher return → leverage makes sense
- **Mixture of risk free asset and optimal portfolio gives better risk/return ratio than just using the stock**
- At the point of tangency 100% is invested in the tangency portfolio
- Risk preferences can be portrayed with utility curves to find an investor's optimal position

Sharpe Ratio - (Brealey, Myers and Allen, 2008)

- $Sharpe\ Ratio = \frac{\overline{r_p} - r_F}{\sigma_p}$
- **Where risk premium = return – risk free rate**
- A measure of a portfolio's risk-return trade-off

Two fund separation theorem

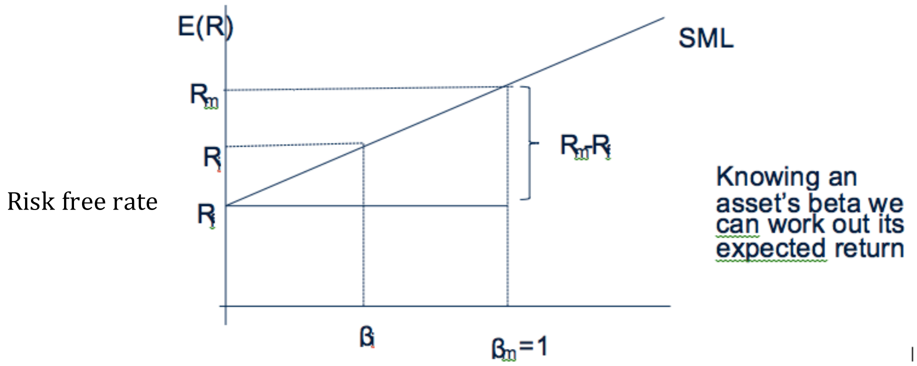
- Theory stating that under conditions in which all investors borrow and lend at the riskless rate, **all investors will choose to possess either a risk free portfolio or the market portfolio (aka Markowitz portfolio theory) – offers the highest Sharpe Ratio**

CAPM

- Rewriting (ri-rf)/(oim/om) = (rm-rf)/om
- (This is the condition for all of the RRRs to be equal to the portfolio RRR)
- $\bar{r}_i = r_F + \beta_{im}(\bar{r}_M - r_F)$
- **ri = risk free rate + (systematic risk)(risk premium)**
- $\beta_{iM} = \frac{\sigma_{iM}}{\sigma_M^2}$
- Investors are only rewarded for risk they must take – no reward for stock specific risk

Securities Market Line (SML)

- Relationship between the asset's return and its market beta is called the SML
- Knowing an asset's beta we can work out its expected return



Market model

- We can decompose an asset's return into three pieces
- $\tilde{r}_i - r_F = \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \tilde{\varepsilon}_i$
- Where $E[\tilde{\varepsilon}_i] = 0$
- $Cov(\tilde{r}_M, \tilde{\varepsilon}_i) = 0$
- Three characteristics of an asset; beta, sigma, alpha

Beta

- Measures systematic risk
- Assets with higher betas are more sensitive to the market

Sigma (e)

- Measures unsystematic risk, not correlated with systematic risk

$$\tilde{r}_i - r_F = \underbrace{\beta_{iM}(\tilde{r}_M - r_F)}_{\text{systematic component}} + \underbrace{\tilde{\epsilon}_i}_{\text{non-systematic component}}$$

$$\underbrace{Var[\tilde{r}_i]}_{\text{total risk}} = \underbrace{\beta_{iM}^2 Var[\tilde{r}_M]}_{\text{systematic risk}} + \underbrace{Var[\tilde{\epsilon}_i]}_{\text{non-systematic risk}}$$

Alpha

- According to CAPM, alpha should be zero for all assets
- Alpha measures an asset's return in excess of its risk adjusted award according to CAPM
- What to do when faced with a positive alpha
    - Check estimation error
    - Past value may not predict its future value
    - Positive may be compensating for other risks

CAPM Summary

- Determines risk-return trade off
- Invest only in risk-free asset and market portfolio, beta measures systematic risk, required rate of return is proportional to beta
- CAPM is simple and sensible
- Built on modern portfolio theory, distinguishes systematic and non systematic risk, provides a simple pricing model
- CAPM is controversial
- Difficult to test, mixed empirical evidence, alternative pricing models might do better

Company cost of capital - (Brealey, Myers and Allen, 2008)
- Expected return on a portfolio of all the company's existing securities
- Opportunity cost of capital for investment in the firm's assets and hence the appropriate discount rate for the firm's average risk projects
- Depends on the use to which that capital is put, provides a general gauge not a specific value
- Project risk is specific to projects and is the most important when considering an investment, measured independently of company cost of capital

Debt and equity - (Brealey, Myers and Allen, 2008)
- Cost of capital a mix of the cost of debt (interest rate) and the cost of equity (expected rate of return demanded by investors in the firm's common stock)
- Cost of debt is less than company cost of capital as debt is safer than the assets
- Cost of equity is greater than the company cost of capital because the equity of a firm that borrows is riskier than the assets as it is a residual claim that stands behind debt
- Company cost of capital = rD (D/D+E) + rE (E/D+E)
- This is called the weighted-average cost of capital or WACC

Equity and asset beta - (Brealey, Myers and Allen, 2008)
- WACC depends on average risk of a company's assets, direct measure called asset beta

Relationship between Asset and Equity Beta

$$\beta_A = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D \qquad (1)$$

The asset (or unlevered) beta β_A equals the weighted average of the equity (or levered) beta β_E and the debt beta β_D. Note that β_A is exogenously determined by the firm's operations and investment projects and thus reflects (non-diversifiable) business risk.
Re-arranging (1) and assuming $\beta_D = 0$, we obtain

$$\beta_E = \frac{E+D}{E}\beta_A \qquad (2)$$

and can see that $\beta_E = \beta_A$ for unlevered firms and that β_E increases with leverage.

CAPM and cost of equity
- $R_e = r_f + (r_m - r_f)B_a + (r_m - r_f)B_a\,(D/E)$
- Re = business risk + financial risk
- Cost of equity a linear function of the leverage ratio $\lambda = D/E$

CAPM and total cost of capital
- Either calculate r_e and plug into WACC formula
- Or calculate $r_e(0)$ on an unlevered firm, which equals r_a as per the 1st MM proposition