TMF1814 DISCRETE MATHEMATICS
TUTORIAL 2
[ANSWERS]

1.3 Functions

1. Find these values.

   We simple round up or down in each case.

   (a) $\lfloor 1.1 \rfloor$ = *1*

   (b) $\lceil 1.1 \rceil$ = ***2***

   (c) $\lceil 2.99 \rceil$ = ***3***

   (d) $\lceil -2.99 \rceil$ = ***-2***

   (e) $\lfloor ½ + \lceil ½ \rceil \rfloor = \lfloor ½ + 1 \rfloor = \lfloor {}^{3}/_{2} \rfloor$ = *1*

   (f) $\lceil \lfloor ½ \rfloor + \lceil ½ \rceil + ½ \rceil = \lceil 0 + 1 + ½ \rceil = \lceil {}^{3}/_{2} \rceil$ = *2*

2. Determine whether $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if

   (a) $f(m,n) = 2m - n$

   This is clearly onto since $f(0, -n) = n$ for any integer n.

   (b) $f(m,n) = m^2 - n^2$

   This is not onto since for example, 2 is not in the range. To see this, if $m^2 - n^2 = (m - n)(m + n) = 2$, then m and n must have same parity (both even or both odd). In either case, both $m - n$ and $m + n$ are then even, so this expression is divisible by 4 and hence cannot equal 2.

   (c) $f(m,n) = |m| - |n|$

   This is onto. To achieve negative values, we set m = 0 and to achieve nonnegative values we set n = 0.

   (d) $f(m,n) = m^2 - 4$

   This is not onto for the same reason as in port (b). in fact, the range here is clearly a subset or the range in that part.

3. Determine whether each of these functions is a bijection from **R** to **R**.

   If we can find an inverse, the function is a bijection. Otherwise we must explain why the function is not one-to-one or not onto.

   (a) $f(x) = -3x + 4$