

Differential Calculus

Tangent Line And Normal Line

The Tangent Line

To find the derivative, one can use its definition which is $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(\Delta x + x) - f(x)}{\Delta x}$. That is to be read as “f prime of x is equal to the limit of f of quantity delta x plus x quantity minus f of x all over delta x as x approaches zero”.

*Derivative - Function that tells the tangent line of the curve; slope

This would be further understood with some examples

Find an equation for the tangent line to the parabola $y = x^2 - 1$ at the point (2,3)

$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(\Delta x + x) - f(x)}{\Delta x}$	Write the definition of derivative
$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(\Delta x + 2) - f(2)}{\Delta x}$	Substitute the x-coordinate, 2, to x
$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{((\Delta x + 2)^2 - 1) - (2^2 - 1)}{\Delta x}$	Substitute to the given equation
$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{((\Delta x)^2 + 4\Delta x + 4 - 1) - (4 - 1)}{\Delta x}$	Expand the square binomial and simplify
$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(\Delta x)^2 + 4\Delta x + 4 - 1 - 4 + 1}{\Delta x}$	Distribute the negative sign
$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(\Delta x)^2 + 4\Delta x}{\Delta x}$	Add common factors
$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta x(\Delta x + 4)}{\Delta x}$	Factor out Δx
$f'(x) = \lim_{\Delta x \rightarrow 0} \Delta x + 4$	Cancel out Δx
$f'(x) = 0 + 4$	Find the limit
$f'(x) = 4$	Simplify

Since the tangent line of any equation is the slope of that same equation, the slope of the equation $y = x^2 - 1$ is 4, which is also its derivative.