We have the following definition for negative exponents. If a is any non-zero number and n is a positive integer (yes, positive) then,

$$a^{-n} = \frac{1}{a^n}$$

Can you see why we required that a not be zero? Remember that division by zero is not defined and if we had allowed a to be zero we would have gotten division by zero. Here are a couple of quick examples for this definition,

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25} \qquad (-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64} = -\frac{1}{64}$$

Here are some of the main properties of integer exponents. Accompanying each property will be a quick example to illustrate its use. We will be looking at more complicated examples after the properties.

Properties

1. $a^n a^m = a^{n+m}$ Example : $a^{-9}a^4 = a^{-9+4} = a^{-5}$
2. $\left(a^n\right)^m = a^{nm}$ Example : $\left(a^7\right)^3 = a^{(7)(3)} = a^{21}$
3. $\dfrac{a^n}{a^m} = \begin{cases} a^{n-m} \\ \dfrac{1}{a^{m-n}} \end{cases}, \quad a \neq 0$ Example : $\dfrac{a^4}{a^{11}} = a^{4-11} = a^{-7}$ or $\dfrac{a^4}{a^{11}} = \dfrac{1}{a^{11-4}} = \dfrac{1}{a^7} = a^{-7}$
4. $(ab)^n = a^n b^n$ Example : $(ab)^{-4} = a^{-4}b^{-4}$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \quad b \neq 0$ Example : $\left(\dfrac{a}{b}\right)^8 = \dfrac{a^8}{b^8}$
6. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$ Example : $\left(\dfrac{a}{b}\right)^{-10} = \left(\dfrac{b}{a}\right)^{10} = \dfrac{b^{10}}{a^{10}}$
7. $(ab)^{-n} = \dfrac{1}{(ab)^n}$ Example : $(ab)^{-20} = \dfrac{1}{(ab)^{20}}$
8. $\dfrac{1}{a^{-n}} = a^n$ Example : $\dfrac{1}{a^{-2}} = a^2$
9. $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$ Example : $\dfrac{a^{-6}}{b^{-17}} = \dfrac{b^{17}}{a^6}$