INMO Mock Test from Delhi INMOTC

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1. Find all $a, b, c \in \mathbb{Q}^+$ such that

$$a + \frac{1}{b}, b + \frac{1}{c}, c + \frac{1}{a} \in \mathbb{Z}$$

2. Find all $f : \mathbb{R} \to \mathbb{R}$ that satisfies

$$f(f(x) + yz) = x + f(y)f(z) \ \forall x, y, z \in \mathbb{R}$$

3. Let *ABC* be a triangle. Points *M* and *N* on ites **GS** *AD*, *AC* respectively satisfy
$$\frac{M}{AB} - \frac{2UN}{AC}$$
The line perpendicular to *MN* passing through *N* intersects *BC* at
R.
Prove that $\angle MFV = 2NC$.

- 4. A subset X of $\{1, 2, ..., n\}$ is interesting if $|X| \in X$ (for a set A, |A| is its cardinality). Find the number of interesting sets with the property that they have no proper interesting subsets.
- 5. Let ABC be an acute angled triangle. A circle passing through A and the triangle's circumcenter O intersects AB and AC at points P and Q, respectively. Prove that the orthocentre of triangle POQ lies on the line BC.
- 6. Prove that for every $n \in \mathbb{Z}_{\geq 0}$, the $7^{7^n} + 1$ is the product of atleast 2n + 3 (not necessarily distinct) primes.

^{*}He made the paper. He is not the one posting it on AoPS