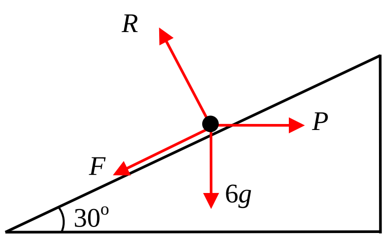
Limiting equilibrium

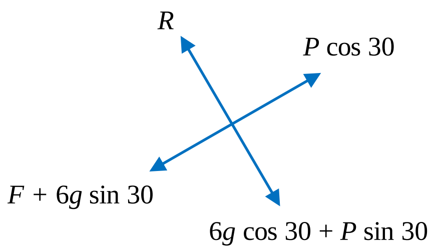
When a particle is in equilibrium but the friction force has reached its maximum or limiting value and the particle is on the point of moving, the particle is said to be in *limiting equilibrium.*

Example: A particle of mass 6 *kg* on a slope of angle 30° is being pushed by a horizontal force of *P N*. If the particle is in *limiting* equilibrium and is on the point of moving up the slope find the value of *P*, given that $\mu = 0{\cdot}3$.

Solution: **DRAW A DIAGRAM SHOWING <u>ALL</u> FORCES**

As the particle is on the point of moving up the slope the friction force will be acting down the slope, and as the particle is in *limiting* equilibrium the friction force will be at its maximum or limiting value, $F = \mu R$.





Res $\nwarrow$ $\Rightarrow$ $R = 6g \cos 30 + P \sin 30$

Limiting equilibrium $\Rightarrow$ $F = \mu R$

$\Rightarrow$ $F = 0{\cdot}3R = 15{\cdot}2766\ldots + 0{\cdot}15P$ **I**

Res $\nearrow$ $\Rightarrow$ $F + 6g \sin 30 = P \cos 30$ **II**

I and **II** $\Rightarrow$ $15{\cdot}2766\ldots + 0{\cdot}15P + 6g \sin 30 = P \cos 30$

$\Rightarrow$ $P = 62{\cdot}395\ldots$

Answer $P = 62$ *N*. to 2 S.F.

Particles connected by pulleys:

The string will always be *inextensible* and *light* and the pulley (or peg) will always be *smooth* or *light*.

1) As the string is *inextensible* the accelerations of the two particles at its ends will have the same magnitude.

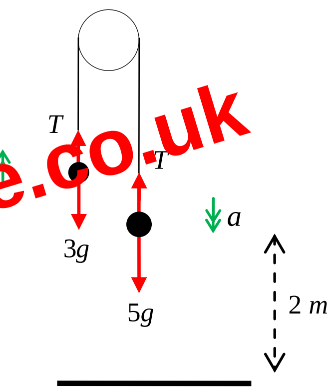
2) As the string is *light*, the tension in the string will be constant along its length.

3) As the pulley (or peg) is *smooth* or *light*, the tensions in the string on either side of the pulley (or peg) will be equal.

Example: Particles of mass 3 *kg* and 5 *kg* are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. The 5 *kg* particle is initially 2 *m* above the floor.

The system is released from rest; find the greatest height of the lighter mass above its initial position in the subsequent motion. Assume that the lighter mass does not reach the pulley.

Solution:



The two particles will move together until the heavier one hits the floor. Then the string will become slack, and the lighter particle will move freely under gravity.

Thus, our problem has **two distinct parts.**

1) What happens **before** the heavy particle hits the floor?

2) What happens **after** the heavy particle hits the floor?

Since the string is inextensible the accelerations of both particles will be equal in magnitude.

Since the string is light and the pulley is smooth the tensions on both sides will be equal in magnitude.

1) For 3 *kg* particle

R $\uparrow$, $F = ma \quad \Rightarrow \quad T - 3g = 3a$ **I**

For 5 *kg* particle

R $\downarrow$, $F = ma \quad \Rightarrow \quad 5g - T = 5a$ **II**

ALWAYS ADD I + II $\quad 2g = 8a \Rightarrow a = 0{\cdot}25g$

Knowing that the acceleration of both particles is $0{\cdot}25g$ $m\ s^{-2}$ we can now find the speed of both particles (equal speeds) when the heavier one hits the floor.

Both particles will have travelled 2 *m* and so, considering the 3 *kg* particle,

$\uparrow+ \quad u = 0,\ a = 0{\cdot}25g,\ s = 2,\ v = ?$ so using $v^2 = u^2 + 2as$

$\Rightarrow \quad v^2 = 2 \times 0{\cdot}25g \times 2 = g \quad \Rightarrow \quad v = \sqrt{g}$

2) The remaining motion takes place freely under gravity as the string will have become slack when the heavier mass hit the floor!

For the 3 *kg* mass

$\uparrow+ \quad u = \sqrt{g}, \quad a = -g, \quad v = 0, \quad s = ?$ so using $v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = g + 2 \times -g \times s \quad \Rightarrow \quad s = 0{\cdot}5$

The 3 *kg* mass travelled 2 *m* before the 5 *kg* mass hit the floor and then moved up a further 0·5 *m* after the string became slack.

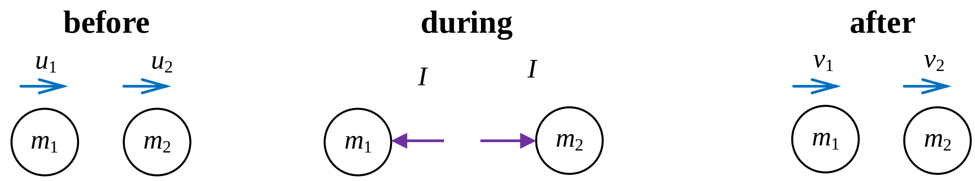
Answer: the lighter particle reached a height of 2.5 *m* above its initial position.

Conservation of linear momentum, CLM.

If there are no external impulses acting on a system then the total momentum of that system is conserved (i.e. remains the same at different times).

or **total momentum before impact equals total momentum after impact.**

Note that if there is an external impulse acting on the system then the momentum *perpendicular* to that impulse is conserved.



$\rightarrow$ + Note that the total impulse on the system is $-I + I = 0$, as the impulses when considering both balls together are *internal*.

$$\boldsymbol{CLM \Leftrightarrow m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2}$$

Example: A railway truck of mass 1500 kg is travelling in a straight line at 3 $m\ s^{-1}$. A second truck of mass 1000 kg is travelling in the opposite direction at 5 $m\ s^{-1}$. They collide (without breaking up) and couple together. With what speed and in what direction are they moving after the impact?

Solution: There is no external impulse (the impulse of gravity is ignored as the time interval is very short) and so momentum is conserved.

ALWAYS DRAW A DIAGRAM – before and **after** (and sometimes ***during***)

You must always choose which direction is positive, then take note of the directions of the arrows in your diagram.

Let the common speed after impact be v ms^{-1} in the direction of the initial velocity of the 1500 kg truck (if this direction is wrong then v will be negative):

$\rightarrow$ + **before** **after**

Taking motion to the right as the positive direction,

CLM

Momentum **before** = $m_1u_1 + m_2u_2 = 1500 \times 3 + 1000 \times (-5) = -500$

Momentum **after** = $m_1v_1 + m_2v_2 = 1500\,v + 1000\,v = 2500\,v$

But momentum is conserved $\Rightarrow$ $-500 = 2500\,v$

$\Rightarrow$ $v = -0{\cdot}2\ m\ s^{-1}$.

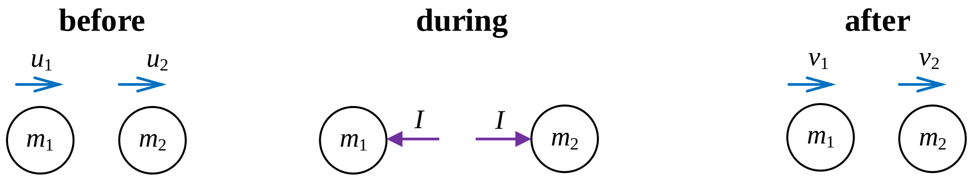
Answer Speed is 0·2 $m\ s^{-1}$ in the direction of the 1000 kg truck's initial velocity.

Appendix

Conservation of linear momentum, C.L.M.

Two balls, masses m_1 and m_2, are moving with speeds u_1 and u_2. They collide and after impact are moving with speeds v_1 and v_2, as shown in the diagram.
There are no external impulses acting on the system

Let the *internal* impulse between the balls be I, acting in opposite directions on each ball.



$\rightarrow +$ using $I = mv - mu$

For m_1: $-I = m_1v_1 - m_1u_1$

For m_2: $I = m_2v_2 - m_2u_2$

Adding: $0 = m_1v_1 + m_2v_2 - m_1u_1 - m_2u_2$

$\Rightarrow$ $\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

or total momentum before collision = total momentum after collision

Thus linear momentum is conserved, giving the **Conservation of Linear Momentum**, **C.L.M.**

Note that if there is an external impulse acting on the system then the momentum *perpendicular* to that impulse is conserved.