

Inverse Trig Functions

$$\begin{aligned} \int \frac{1}{\sqrt{a^2 - u^2}} du &= \sin^{-1}\left(\frac{u}{a}\right) + c & \int \sin^{-1} u \, du &= u \sin^{-1} u + \sqrt{1 - u^2} + c \\ \int \frac{1}{a^2 + u^2} du &= \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + c & \int \tan^{-1} u \, du &= u \tan^{-1} u - \frac{1}{2} \ln(1 + u^2) + c \\ \int \frac{1}{u\sqrt{u^2 - a^2}} du &= \frac{1}{a} \sec^{-1}\left(\frac{u}{a}\right) + c & \int \cos^{-1} u \, du &= u \cos^{-1} u - \sqrt{1 - u^2} + c \end{aligned}$$

Hyperbolic Trig Functions

$$\begin{aligned} \int \sinh u \, du &= \cosh u + c & \int \operatorname{sech} u \tanh u \, du &= -\operatorname{sech} u + c & \int \operatorname{sech}^2 u \, du &= \tanh u + c \\ \int \cosh u \, du &= \sinh u + c & \int \operatorname{csch} u \coth u \, du &= -\operatorname{csch} u + c & \int \operatorname{csch}^2 u \, du &= -\coth u + c \\ \int \tanh u \, du &= \ln(\cosh u) + c & \int \operatorname{sech} u \, du &= \tan^{-1}|\sinh u| + c \end{aligned}$$

Miscellaneous

$$\begin{aligned} \int \frac{1}{a^2 - u^2} du &= \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + c & \int \frac{1}{u^2 - a^2} du &= \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + c \\ \int \sqrt{a^2 + u^2} \, du &= \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln \left| u + \sqrt{a^2 + u^2} \right| + c \\ \int \sqrt{u^2 - a^2} \, du &= \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + c \\ \int \sqrt{a^2 - u^2} \, du &= \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{u}{a}\right) + c \\ \int \sqrt{2au - u^2} \, du &= \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + c \end{aligned}$$

Standard Integration Techniques

Note that all but the first one of these tend to be taught in a Calculus II class.

u Substitution

Given $\int_a^b f(g(x))g'(x)dx$ then the substitution $u = g(x)$ will convert this into the integral, $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u) du$.

Integration by Parts

The standard formulas for integration by parts are,

$$\int u dv = uv - \int v du \qquad \int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

Choose u and dv and then compute du by differentiating u and compute v by using the fact that $v = \int dv$.