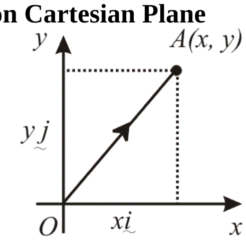
$$\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$$

Parallel Vectors

$\overrightarrow{AB}$ is parallel to $\overrightarrow{PQ}$ if $\dfrac{\overrightarrow{AB}}{\overrightarrow{PQ}}=k$, where k is a constant.

If $\dfrac{\overrightarrow{AB}}{\overrightarrow{BC}}=k$, since B is a common point, A, B and C are collinear.

Vector on Cartesian Plane



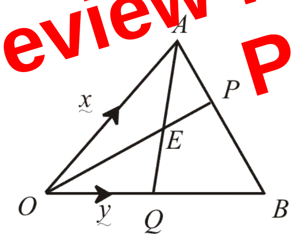
$\overrightarrow{OA}=x\underset{\sim}{i}+y\underset{\sim}{j}$

$\left|\overrightarrow{OA}\right|=\sqrt{x^2+y^2}$ = magnitude of vector OA

Unit vector in the direction of

$\overrightarrow{OA}=\dfrac{x\underset{\sim}{i}+y\underset{\sim}{j}}{\sqrt{x^2+y^2}}$

Example :



Given $\overrightarrow{OA}$ = $\underset{\sim}{x}$ and $\overrightarrow{OB}=\underset{\sim}{y}$. P is a point on AB such that AP : PB = 1 : 2 and Q is the midpoint of OB. The line OP intersects AQ at the point E. Given $\overrightarrow{OE}$ = k$\overrightarrow{OP}$ and $\overrightarrow{AE}=h\overrightarrow{AQ}$, where h and k are constants,

(a) find $\overrightarrow{OQ}$ and $\overrightarrow{OP}$ in terms of $\underset{\sim}{x}$ and/or $\underset{\sim}{y}$.

(b) Express $\overrightarrow{OE}$ in terms of
  (i) k, $\underset{\sim}{x}$ and $\underset{\sim}{y}$,
  (ii) h, $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

(c) Hence, find the value of h and k.

(a) $\overrightarrow{OQ}$ = $\dfrac{1}{2}\overrightarrow{OB}=\dfrac{1}{2}\underset{\sim}{y}$

$\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}$ = $\underset{\sim}{x}+\dfrac{1}{3}\overrightarrow{AB}$

= $\underset{\sim}{x}+\dfrac{1}{3}(\underset{\sim}{y}-\underset{\sim}{x})$ = $\dfrac{1}{3}(2\underset{\sim}{x}+\underset{\sim}{y})$

(b) (i) $\overrightarrow{OE}$ = k$\overrightarrow{OP}$ = k × $\dfrac{1}{3}(2\underset{\sim}{x}+\underset{\sim}{y})$

= $\dfrac{2k}{3}\underset{\sim}{x}+\dfrac{k}{3}\underset{\sim}{y}$

(ii) $\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}$ = $\overrightarrow{OA}$ + h$\overrightarrow{AQ}$

= $\underset{\sim}{x}+h(-\underset{\sim}{x}+\dfrac{1}{2}\underset{\sim}{y})$

= (1 – h)$\underset{\sim}{x}$ + $\dfrac{h}{2}\underset{\sim}{y}$

(c) Compare the coefficient of $\underset{\sim}{x}$ and $\underset{\sim}{y}$

1 – h = $\dfrac{2k}{3}$ ---------------(1)

and $\dfrac{h}{2}=\dfrac{k}{3}$, h = $\dfrac{2k}{3}$ -----(2)

Substitute in (1)

$1-\dfrac{2k}{3}=\dfrac{2k}{3}$

$1=\dfrac{4k}{3}$ ∴ k = $\dfrac{3}{4}$

h = $\dfrac{2k}{3}$ = $\dfrac{2}{3}$ × $\dfrac{3}{4}$ = $\dfrac{1}{2}$.

Example :

Given $\overrightarrow{OP}=\begin{pmatrix}3\\-4\end{pmatrix}$ and $\overrightarrow{OQ}=\begin{pmatrix}1\\5\end{pmatrix}$.

(a) Find $\left|\overrightarrow{OP}\right|$

(b) Find the unit vector in the direction of $\overrightarrow{OP}$.

(c) Given $\overrightarrow{OP}$ = m$\overrightarrow{OA}$ – n $\overrightarrow{OQ}$ and A is the point (–2, 7). Find the value of m and n.

(a) $\left|\overrightarrow{OP}\right|$ = $\sqrt{3^2+(-4)^2}=\sqrt{25}=5$

(b) Unit vector in the direction of $\overrightarrow{OP}$ = $\dfrac{3i-4j}{5}$

(c) $\overrightarrow{OP}$ = m$\overrightarrow{OA}$ – n $\overrightarrow{OQ}$

$$\begin{pmatrix}3\\-4\end{pmatrix}=m\begin{pmatrix}-2\\7\end{pmatrix}-n\begin{pmatrix}1\\5\end{pmatrix}$$