CHAPTER-10 (VECTOR ALGEBRA)

(1 mark)

- 1. Find the cosine of an acute angle between the vectors $2\hat{i} 3\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} 2\hat{k}$.
- 2. If $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} \mathbf{b}|$ then find the angle between \mathbf{a}^* and \mathbf{b}^* .
- 3. If $|\vec{a}| = 5$, $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$. Find $\vec{a} \cdot \vec{b}$.
- 4. Find the position vector of the mid point of the vector joining the points P(2,3,4) and Q(4,1,-2).
- 5. Find the value of 'x' for which x(i + j + k) is a unit vector.
- 6. If the position vector \mathbf{a} of the point (5,n) is such that $|\mathbf{a}| = 13$, find the value of n.
- 7. If the vector $\mathbf{a}^* = 2\mathbf{i} 3\mathbf{j}$ and $\mathbf{b} = -6\mathbf{i} + m\mathbf{j}$ are collinear, find the value of m.
- 8. If a vector makes angles α , β , γ with x-axis, y-axis, z-axis respectively, then what is the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.
- 9. If $|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2$, what is the angle between \mathbf{a} and \mathbf{b} ?

10. If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, then find $|\vec{x}|$.



(6 Marks)

- 1. Prove that $2(\vec{a} \times \vec{b}) = (\vec{a} \vec{b}) \times (\vec{a} + \vec{b})$. Also give its geometrical inter pretation. Hence find the area of a parellogram whose diagonals are the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$
- 2. Let \vec{a}, \vec{b} and \vec{c} be unit vectors, such that $\vec{a}.\vec{b}.=\vec{a}\vec{c}=0$ and the angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$. Prove that $\vec{a}=\pm 2\mathbf{O}\times\vec{c}$
- 3. A line makes angles $\mathbf{a}, \mathbf{b}, \zeta$ and δ with the for diagenals of a cube. Prove that

$$\cos^2 \mathbf{a} + \cos^2 \mathbf{b} + \cos^2 \zeta + \cos^2 \delta = \frac{4}{3}$$

4. If $\vec{a}.\vec{b}$ and \vec{c} are three mutually perpendicular vectors of equal magnitude, find the angle between \vec{a} and $(\vec{a} + \vec{b} + \vec{c})$

5. Let $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express \vec{b} in the form of $\vec{b} = \vec{c} + \vec{d}$. Where \vec{c} is parallel to \vec{a} and \vec{d} is perpendicular to \vec{a} .

Phosphorous	0.75	1.50
Iron	1.60	0.80

The minimum requirement of the nutrients in the diet are 1.00 mg of thiamine, 7.50 mg of phosphorous and 10.00 mg of iron. The cost of F_1 is 20 paise per 25 gms while the cost of F_2 is 15 paise per 25 gms. Find the minimum cost of diet.

- 11. A farmer is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it is necessary to buy two additional products, say, A and B. One unit of products A contains 36 units of X, 3 units of Y, and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirements of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs.20 per and product B costs Rs.40 per unit. Formulate the above as a linear programming problem to minimize the total cost, and solve the problem by using graphical method.
- 12. If a youngman rides his motor cycle at 25 km per hour he has to spend Rs 2 per km. on petrol, if he rides it at a faster speed of 40 km. per hour, the petrol cost increase to Rs. 5 per km. He has Rs. 100 to spend or petrol, and wish to find what is the maximum distance he can travel with the neurone hour. Express this as a linear programming problem and the neuron graphically.
- 13. An aeroplane can carry a maximum 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 306 is made on each economy class ticket a the arrine reserves at mast 20 seats for first class. However at least of the arrine reserves at mast 20 seats for first class. However at class. Determine reserves at mast 20 seats must be sold in order to maximize the profit for the airline. What is the maximum profit?
- 14. Two godowns, A and B, have a grain storage capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops D, E, F whose requirements are 60, 50, 40 quintals, respectively. The costs of transportation per quintal from godowns to the shops are given in the following table :

From	Transportation cost per quintal	
То	Α	В
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum ?

15 An oil company has two depots, A and B, with capacities of 7000L and 4,000L respectively . The Company is to supply oil to three petrol pumps D,E, F whose

- 9. Find the probability distribution of the number of face cards when two cards are drawn without replacement from a well shuffled deck of 52 cards.
- **10.** Find the probability distribution of number of doublets in 4 throws of a pair of dice.
- 11. Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?
- 12. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of the cases are they likely to contradict each other in the stating the same fact?
- 13. A factory has three machines I,II and III which produce 30%, 50% and 20% respectively of the total items of the same variety. Out of these 2%, 5% and 3% respectively are found to be defective. An item is picked up at random and found to be defective. Find the probability that it is produced by the III machine.

Ans.:-6/37

Ans. n= 10

14. A random variable X has the following probability distribution:-



problem independently A and B are $\frac{1}{2}$ and $\frac{1}{2}$

respectively. If both try to solve the problem independently. Find the

probability that

- i. The problem is solved
- ii. Exactly one of them solves the problem.

18. A bag contains (2n+1) coins. It is known that n of these coins have a head on both sides where the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$. Determine the value of n.

19.A ship is fitted with three engines E_1 , E_2 and E_3 . The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and X_1 , X_2 and X_3 denote respectively the events that the

engines E_1 , E_2 and E_3 are functioning then find $P\left(\frac{X}{X_1}\right)$. Ans. $\frac{7}{16}$