"ALL What we need to know" "about ax^2+bx+c"

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$$y = ax^2+bx+c = f(x), \quad (a \neq 0).$$

(A) Completing the Square / "History of Delta"

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$$= a\Big[\underbrace{(x)^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2}_{\text{We complete the square}}+\frac{c}{a}\Big]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c(4a)}{a(4a)}\right]$$

$$\begin{cases} ax^2+bx+c = a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right] \\ \text{for all } x \in \mathbb{R}. \end{cases} \quad (1)$$

Res (1)

$$\begin{cases} ax^2+bx+c = \boxed{a(x-\alpha)^2+\beta} \quad (2) \\ \text{with} \quad \alpha = \frac{-b}{2a} \text{ and } \beta = \frac{4ac-b^2}{4a} \end{cases}$$

Rem (1), We denote $\Delta = \text{Delta} = b^2-4(a)(c)$

$$\boxed{ax^2+bx+c = a(x-\alpha)^2+\frac{-\Delta}{4a}} \quad (3)$$

Res (2),

$\boxed{a>0}$

$$\begin{cases} ax^2+bx+c \geqslant \frac{-\Delta}{4a} \\ \text{for all } x \in \mathbb{R}. \end{cases}$$

$\boxed{a<0}$

$$\begin{cases} ax^2+bx+c \leqslant \frac{-\Delta}{4a} \\ \text{for all } x \in \mathbb{R}. \end{cases}$$

Proof.

Since: $\cancel{x+\frac{b}{2a}}$ $(x-\alpha)^2 \geqslant 0$

and $a>0 \Rightarrow a(x-\alpha)^2 \geqslant 0$

$\Rightarrow a(x-\alpha)^2+\frac{-\Delta}{4a} \geqslant 0+\frac{-\Delta}{4a}$

Since $a<0$

$\Rightarrow a(x-\alpha)^2 \leqslant 0$

$a(x-\alpha)^2+\frac{-\Delta}{4a} \leqslant \frac{-\Delta}{4a}$