$$\beta(m,n) = \int_0^{\pi/2} \sin^{2m-2}\theta \cos^{2n-1}\theta \, d\theta$$

(p) (q)

$$\beta(m,n) = \int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

Result:

The expression of $\beta(m,n)$ with $0,\infty$ as limits

$$\beta(m,n) = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}} \, dy$$

Result:

$$\Gamma(n)\,\Gamma(1-n) = \beta(n, 1-n) = \frac{\pi}{\sin n\pi}$$

eg 1 Evaluate the following using gamma function:

(a) $\int_0^{\infty} e^{-x^2} \, dx$

put $x^2 = t$

$2x \, dx = dt \Rightarrow dx = \frac{dt}{2\sqrt{t}}$

$t : 0 \to \infty$