2 INVERSE TRIGONOMETRIC FUNCTIONS

KEY CONCEPT INVOLVED

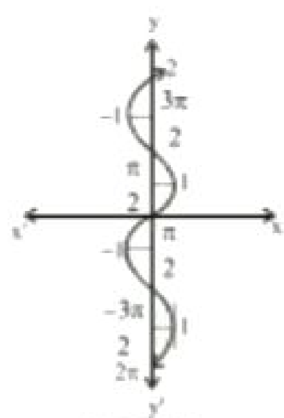
1.

	Functions	**Domain**	**Range**
(i)	sin	R	[−1, 1]
(ii)	cos	R	[−1, 1]
(iii)	tan	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in z\}$	R
(iv)	cot	$R - \{x : x = n\pi, n \in z\}$	R
(v)	sec	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in z\}$	$R - [-1, 1]$
(vi)	cosec	$R - \{x : x = n\pi, n \in z\}$	$R - [-1, 1]$

2. Inverse Function - If $f : X \to Y$ such that $y = f(x)$ is one-one and onto then we define another function $g : Y \to X$ such that $x = g(y)$, where $x \in X$ and $y \in Y$ which is also one-one and onto. In such a case domain of g = Range of f and Range of g = domain of f
g is called inverse of f or $g = f^{-1}$
Inverse of $g = g^{-1} = (f^{-1})^{-1} = f$

3. Principal value Branch of function $\sin^{-1}$ - It may be noted that for the domain [-1, 1] the range sould be any one of the intervals $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ or $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ corresponding to each interval we get a branch of the function $\sin^{-1}$ the branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch.

Thus $\sin^{-1} : [-1, 1] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$



$y = \sin^{-1} x$

In particualr $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Prop. V $\int_0^{2a} f(x)\,dx$

Prop. V $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f(x) is even function

$\int_{-a}^{a} f(x)\,dx = 0$, if f(x) is odd function

Prop. VI $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

Prop. VII $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(2a-x) = f(x)$

$\int_0^{2a} f(x)\,dx = 0$, if $f(2a-x) = -f(x)$

9. Definite Integral as the limit of a sum

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\,[f(a) + f(a+h) + f(a+2h) + \cdots + f< a + (n-1)h)]$$

or $$\int_a^b f(x)\,dx = \lim_{h\to 0} h\,[f(a+h) + f(a+2h) + f(a+3h) + \cdots + f(a+nh)]$$

where, $$h = \frac{b-a}{n}$$

$\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)\,dt = f\{v(x)\}\frac{d}{dx}v(x) - f\{u(x)\}\frac{d}{dx}u(x)$ this rule is called Leibnitz's is Rule.

CONNECTING CONCEPTS

1. Integration is an operation on functions.

2. $\int [k_1 f_1(x) + k_2 f_2(x) + \ldots\ldots + k_n f_n(x)]\,dx$

$= k_1\int f_1(x)\,dx + k_2\int f_2(x)\,dx + \ldots\ldots + k_n\int f_n(x)\,dx$

3. All functions are not integrable and the integral of a function is not unique.

4. If a polynomial function of a degree n is integrated we get a polynomial of degree n + 1

4. **Integration by using standard formulae –**

1. $\int k\,dx = kx + c$, k is constant
2. $\int k f(x)\,dx = k\int f(x)\,dx + c$
3. $\int (f_1(x) \pm f_2(x))\,dx = \int f_1(x)\,dx \pm \int f_2(x)\,dx + c$
4. $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ (n \neq -1)$
5. $\int \frac{1}{x}\,dx = \log_e |x| + c$
6. $\int a^x dx = \frac{a^x}{\log_e a} + c, a > 0$
7. $\int e^x dx = e^x + c$