Electromagnetic Formula Sheet

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Vector-Analysis:

Elements	Rectangular	Cylindrical	Spherical
dl	$dxa_x + dya_y + dza_z$	$dra_r + rd\emptyset a_\emptyset + dza_z$	$dRa_R + R\,d\theta a_\theta + R\sin\theta d\emptyset a_\emptyset$
dS	$dydza_x + dzdxa_y + dxdya_z$	$rd_\emptyset dza_r + drdza_\emptyset + rdrd_\emptyset a_z$	$R^2\sin\theta d\emptyset d\theta a_R + R\sin\theta d\emptyset dRa_\theta + RdRd\theta a_\emptyset$
dV	$dxdydz$	$rdrd_\emptyset dz$	$R^2\sin\theta d\emptyset d\theta dR$

Cylindrical to Rectangular and vice versa

$$\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix} = \begin{bmatrix}cos\emptyset & -sin\emptyset & 0\\ sin\emptyset & cos\emptyset & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}Ar\\A\emptyset\\Az\end{bmatrix}, \quad \begin{bmatrix}Ar\\A\emptyset\\Az\end{bmatrix} = \begin{bmatrix}cos\emptyset & sin\emptyset & 0\\ -sin\emptyset & cos\emptyset & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix},$$

$x = rcos\emptyset, y = rsin\emptyset, r = x^2 + y^2,$ $\emptyset = tan\frac{y}{x}$

Spherical to Rectangular and vice versa

$$\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix} = \begin{bmatrix}sin\theta cos\emptyset & cos\theta cos\emptyset & -sin\emptyset\\ sin\theta sin\emptyset & cos\theta sin\emptyset & cos\emptyset\\ cos\theta & -sin\theta & 0\end{bmatrix}\begin{bmatrix}AR\\A\theta\\A\emptyset\end{bmatrix}, \quad \begin{bmatrix}AR\\A\theta\\A\emptyset\end{bmatrix} = \begin{bmatrix}sin\theta cos\emptyset & sin\theta sin\emptyset & cos\theta\\ cos\theta cos\emptyset & cos\theta sin\emptyset & -sin\theta\\ -sin\emptyset & cos\emptyset & 0\end{bmatrix}\begin{bmatrix}Ax\\Ay\\Az\end{bmatrix},$$

$x = Rsin\theta cos\emptyset, y = Rsin\theta cos\emptyset,\ z = Rcos\vartheta, R = x^2 + y^2 + z^2, \theta = tan^{-1}(R/\sqrt{(x^2+y^2)})$

Coordinate Systems	Gradient	divergence
Rectangular	$\nabla V = \frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z$	$\nabla . A = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$
Cylindrical	$\nabla V = \frac{\partial V}{\partial r}a_r + \frac{\partial V}{r\partial\emptyset}a_\emptyset + \frac{\partial V}{\partial z}a_z$	$\nabla . A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial}{r\partial\emptyset}A_\emptyset + \frac{\partial}{\partial z}A_z$
Spherical	$\nabla V = \frac{\partial V}{\partial R}a_R + \frac{\partial V}{R\partial\theta}a_\theta + \frac{\partial V}{Rsin\theta\partial\emptyset}a_\emptyset$	$\nabla V = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2A_R) + \frac{1}{Rsin\theta}\frac{\partial}{R\partial\theta}(A_\theta sin\theta) + \frac{\partial}{Rsin\theta\partial\emptyset}A_\emptyset$

Electrostatics:

$F = \frac{Q_1Q_2}{4\pi\epsilon_o R^2}a_{R12}$, $F = qE$, $\nabla\cdot E = \frac{\rho_v}{\epsilon_o}$, $\nabla\times E = 0$, $\oint_S E.ds = \frac{Q}{\epsilon_o}(gauss's\ law)$, $\oint_C E.dl = 0$, $E = \frac{q}{4\pi\epsilon_o R^2}a_R$, $p = qd$, $E = \frac{1}{4\pi\epsilon_o}\int_V \frac{\rho}{R^2}dv\ a_R$, $E = \frac{1}{4\pi\epsilon_o}\int_V \frac{\rho\mathbf{R}}{R^3}dv$, $E = \frac{1}{4\pi\epsilon_o}\int_S \frac{\rho_s}{R^2}ds\ a_R$, $E = \frac{1}{4\pi\epsilon_o}\int_L \frac{\rho_l}{R^2}dl\ a_R$, $E = \frac{\rho_l}{2\pi\epsilon_o r}a_r$, $E = \frac{\rho_s}{2\epsilon_o}a_n$, $E = -\nabla V$, $V = \frac{q}{4\pi\epsilon_o R}$, $V = \frac{W}{Q} = -\int_A^B E.dl$, $D = \epsilon_o E$, $Q = \oint_S D\cdot ds = \int\rho_v dv$, $\nabla\cdot D = \rho_v$, $V_{dipole} = \frac{Qdcos\theta}{4\pi\epsilon R^2}$, $Q = CV$, $C = \frac{\epsilon s}{d}$, $C = \frac{2\pi\epsilon L}{ln(\frac{b}{a})}(for\ cylindrical\ capacitor)$, $\nabla^2 V = -\frac{\rho}{\epsilon}(poisson's\ equation)(\nabla^2\ is\ laplacian\ operator)$, $\nabla^2 V = 0(Laplace\ equation)(no\ charge\ is\ present\ \rho = 0)$

Magnetostatics:

$I = \frac{\Delta Q}{\Delta t}$, $I = \int J\cdot ds$, $J = \sigma E(A/m^2)(OHM's\ law\ in\ waveform)$, $\nabla . J = -\frac{\partial\rho}{\partial t}(Equation\ of\ continuity)(A/m^3)$, $\nabla . J = 0(for\ steady\ current\ divergenceless)$, $\oint_S J\cdot ds = 0$, $\nabla\times\left(\frac{J}{\sigma}\right) = 0$, $\oint_C \frac{1}{\sigma}J\cdot dl = 0$, $F_m = q(\mathbf{V}\times\mathbf{B})$,

$F = F_m + F_E = q(\mathbf{E} + \mathbf{V}\times\mathbf{B})(lorentz's\ force\ equation)$, $\nabla . B = 0$, $\nabla\times B = \mu_o J$, $\oint_S B.ds = 0$(law of conservation of magnetic flux), $\oint_C B\cdot dl = \mu_o I$, $B = \nabla\times A$, $\nabla\cdot A = 0$, $\nabla^2 A = -\mu_o J$, $A = \frac{\mu_o I}{4\pi}\oint_C \frac{dl'}{R}$, $B = \frac{\mu_o I}{4\pi}\oint_C \nabla\times\left(\frac{dl'}{R}\right)$, $B = \frac{\mu_o I}{4\pi}\oint_C\left(\frac{dl'\times a_R}{R^2}\right)$, $B = \frac{\mu_o I}{4\pi}\oint_C\left(\frac{dl'\times\mathbf{R}}{R^3}\right)$, $B = \mu H$, $M = \chi_m H$, $1 + \chi_m = \mu_r$, $\oint_C H\cdot dl = I\ (Ampere's\ circuital\ law)$, $\Phi = \oint_C A\cdot dl = \oint_S B\cdot ds$, $B_\emptyset = \frac{\mu_o I}{2\pi r}a_\emptyset(line\ of\ current)$, $a_\emptyset = a_l\times a_r$, $\Phi_{12} = L_{12}I_1$, $\Lambda_{12} = N_2\Phi_{12} = L_{12}I_1$, $F_m = I\int dl\times B$

Time Varying Fields and Maxwell's Equations:

$V_{emf} = -\frac{d\Phi}{dt}$, $V_{emf} = \oint E\cdot dl = -\int\frac{\partial B}{\partial t}\cdot ds$, $V_{emf} = -\oint(u\times B)\cdot dl$,

Maxwell equations

Differential form	Integral form	Significance
$\nabla\times E = -\frac{\partial B}{\partial t}$	$\oint E\cdot dl = -\frac{d\Phi}{dt}$	Faraday's Law
$\nabla\times H = J + \frac{\partial D}{\partial t}$	$\oint H\cdot dl = I + \oint_S\frac{\partial D}{\partial t}\cdot ds$	Ampere's circuital Law
$\nabla\cdot D = \rho_v$	$\oint D\cdot ds = Q$	Gauss's Law
$\nabla . B = 0$	$\oint B\cdot ds = 0$	No isolated magnetic charge