Joint PDF of two continuous Random Variables: $f_{X,Y}(x\,,y) = \frac{\partial^2 F_{X,Y}(x\,,y)}{\partial x \partial y}$

Marginal pdf: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x\,,y')dy'$ and $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(\,x',y)dx'$

Properties:

Let $f_{X,Y}(x\,,y) = \begin{cases} 1, & 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$

1. If x<0 or y<0, then CDF $F_{X,Y}(x,y) = 0$
2. If (*x*, *y*) is inside the unit interval,

$$F_{X,Y}(x,y) = \int_0^x \int_0^y 1dx'dy' = xy$$

3. Similarly, If $0 \le y \le 1$ and x>1, $F_{X,Y}(x,y) = y$
4. If $0 \le x \le 1$ and y>1,

$$F_{X,Y}(x,y) = \int_0^x \int_0^1 1dx'dy' = x$$

5. Finally if x>1 and y>1,

$$F_{X,Y}(x,y) = \int_0^1 \int_0^1 1dx'dy' = 1$$

Independence of two variables:

if X and Y are independent discrete random variables, then the joint pmf is equal to the product of the marginal pmf's.

$p_{X,Y}(x_j\,,y_k) = p_X(x_j\,)p_Y(y_k)\,,$ for all x_j and y_k

$F_{X,Y}(x\,,y) = F_X(x\,)F_Y(y)\,,$ for all x and y

$f_{X,Y}(x\,,y) = f_X(x\,)f_Y(y)\,,$ for all x and y

Expected Value of a Function of Two Random Variables:

$$E[Z] = \begin{cases} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{X,Y}(\,x,y)dxdy\,, & \text{X, Y jointly continuous} \\ \sum_i \sum_n g(x_i\,,y_n) p_{X,Y}(x_i\,,y_n)\,, & \text{X, Y discrete} \end{cases}$$

Joint moment of *X* and *Y*:

$$E[X^j Y^k] = \begin{cases} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^j y^k f_{X,Y}(x,y)dx\,, & \text{X, Y jointly continuous} \\ \sum_i \sum_n x_i^j y_n^k p_{X,Y}(x_i\,,y_n)\,, & \text{X, Y discrete} \end{cases}$$

If E[XY]=0, then we say that *X* and *Y* are orthogonal.

Covariance of *X* and *Y*: COV(X,Y)=E[XY]-E[X]E[Y]

Pairs of independent random variables have covariance zero.

Correlation coefficient of *X* and *Y*:

$$\rho_{X,Y} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}, \text{ where } \sigma_X = \sqrt{VAR(X)} \text{ and } \sigma_Y = \sqrt{VAR(Y)}$$

X and *Y* are said to be uncorrelated if $\rho_{X,Y} = 0$

Conditional Probability:

$$p_Y(y|x\,) = \frac{p_{X,Y}(x,y)}{p_X(x)} \qquad F_Y(y|x_k\,) = \frac{P[Y \le y, X = x_k]}{P[X = x_k]} \qquad f(y|x_k\,) = \frac{d}{dy} F_Y(y|x_k\,)$$

Conditional Expectation:

$$E[Y|x] = \int_{-\infty}^{\infty} y f_Y(\,y|x)dy \qquad E[Y|x_k] = \sum_{y_j} y_j p_Y(\,y_j|x)$$

Central Limit Theorem

Let S_n be the sum of n iid random variables with finite mean E[X]=μ and finite variance σ^2 and let Z_n be the zero-mean, unit-variance random variable defined by

$$Z_n = \frac{S_n - n\mu}{\sigma\sqrt{n}}$$