

* EUCLIDEAN NORM (l_{∞})

l_{∞} norm is called the Euclidean norm of the vector x because it represents usual notion of distance from the origin.

Let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ then $\|x\|_{\infty} = \max_{1 \leq i \leq n} |x_i|$
[we don't see signs, but see magnitudes]
eg: $\{1, -2, 3, -4\}$ then $\|x\|_{\infty} = 4$

* GAUSS JACOBI METHOD

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} A = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & a_{32} & 0 \end{bmatrix} L + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} D + \begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & 0 & a_{23} \\ 0 & 0 & 0 \end{bmatrix} U$$

$$x^{(k+1)} = D^{-1} [B - Lx^{(k)} - Ux^{(k)}] = [B - Lx^{(k)} - Ux^{(k)}] / D$$

$$\Rightarrow \begin{aligned} x_1^{(k+1)} &= [b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}] / a_{11} \\ x_2^{(k+1)} &= [b_2 - a_{21}x_1^{(k)} - a_{23}x_3^{(k)}] / a_{22} \\ x_3^{(k+1)} &= [b_3 - a_{31}x_1^{(k)} - a_{32}x_2^{(k)}] / a_{33} \end{aligned}$$

* GAUSS SEIDEL METHOD

(use the updated value, otherwise same as G-Jacobi Method)

$$x^{(k+1)} = D^{-1} [B - Lx^{(k+1)} - Ux^{(k)}]$$

$$x_1^{(k+1)} = [b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}] / a_{11}$$

$$x_2^{(k+1)} = [b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}] / a_{22}$$

$$x_3^{(k+1)} = [b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}] / a_{33}$$

* SOR METHOD

Let we have the solution at k^{th} iteration i.e. $x^{(k)}$ then we find $x^{(k+1)}$ using G-seidel method. Then SOR solution at $(k+1)^{\text{th}}$ iteration as given by $x^{(k+1)} = (1-\omega)x^{(k)} + \omega x_{\text{GS}}^{(k+1)}$
where $\omega \rightarrow$ relaxation parameter, $1 \leq \omega \leq 2$