Signals & Systems Formula Sheet *by Haris H.*

Geometric Series formulas

Interval	Sum	Condition	Interval	Sum	Condition
Infinite	$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$	|a|<1	***Finite on [1,N]***	$\sum_{k=1}^{N} a^k = \frac{a(1-a^{N+1})}{1-a}$	None
Finite on [0,N]	$\sum_{k=0}^{N} a^k = \frac{1-a^{N+1}}{1-a}$	None	***Finite on [N₁,N₂]***	$\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1}-a^{N_2+1}}{1-a}$	None
Infinite	$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}$	|a|<1	***Finite on [1,N]***	$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$	None

Elementary Signals classification

Name	Continuous	Discrete	Name	Continuous	Discrete
Unit Step function	$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$	**Signum signal**	$Sgn(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$	$Sgn[n] = \begin{cases} 1, & n > 0 \\ -1, & n < 0 \end{cases}$
Ramp signal	$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$	$r[n]=nu(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$	**Sinusoidal signal**	$x(t) = \sin(2\pi f_0 t + \theta)$	$X[n] = \sin(2\pi f_0 n + \theta)$
Impulse function	$\delta(t) = 0, t \neq 0$	$\delta[n] = \begin{cases} 1, n = 0 \\ 0, otherwise \end{cases}$	**Sinc function**	$\text{sinc}(\omega_0 t) = \frac{\sin(\pi\omega_0 t)}{\pi\omega_0 t}$	$\text{sinc}[\omega_0 n] = \frac{\sin(\pi\omega_0 n)}{\pi\omega_0 n}$
Rectangular pulse function	$\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1, \|t\| \leq \tau/2 \\ 0, \|t\| > \tau/2 \end{cases}$	$\Pi[\frac{n}{2N}] = \begin{cases} 1, \|n\| \leq N \\ 0, \|n\| > N \end{cases}$	**Triangular pulse**	$\Lambda\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \|\frac{t}{\tau}\|, t \leq \|\tau\| \\ 0 \quad, t > \|\tau\| \end{cases}$	$\Lambda[\frac{n}{N}] = \begin{cases} 1 - \frac{\|n\|}{N}, \|n\| \leq N \\ 0 \quad, elsewhere \end{cases}$

Important Properties of Signals

Name	Properties	Name	Properties
Signals in term of unit step and vice versa	$r(t) = tu(t)$ $u(t) = \frac{d}{dt} r(t)$ $\delta(t) = \frac{d}{dt} u(t)$ $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$ $sgn = u(t) - u(-t)$ $sgn = 2u(t) - 1$ $\Pi\left(\frac{t}{\tau}\right) = u\left(t + \frac{t}{\tau}\right) - u(t - \frac{t}{\tau})$	***Impulse properties***	$\int_{-\infty}^{\infty} \delta(t) dt = 1$ $\delta(\alpha t) = \frac{1}{\|\alpha\|}\delta(t)$ $\delta(\alpha t + b) = \frac{1}{\|\alpha\|}\delta(t + \frac{b}{\alpha})$ $\int_{-\infty}^{\infty} \emptyset(t)\delta(t-\lambda) dt = \emptyset(\lambda)$ $\emptyset(t)\delta(t-\lambda) = \emptyset(\lambda)\delta(t-\lambda)$
Time period of linear combination of two signals	Sum of signals is periodic if $\frac{T_1}{T_2} = \frac{m}{n}$ = rational number The fundamental period of g(t) is given by nT1 = mT2 provided that the values of m and n are chosen such that the greatest common divisor (gcd) between m and n is 1	**Odd and even & symmetry**	$x_e(t) = x_e(-t)$ $x_o(t) = -x_o(-t)$ $x(t) = x_e(t) + x_o(t)$ $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$
Combined operation	*x(t)⇒ Kx(t) +C* Scale by *K* then shift by *C ….* x(t)⇒ x(αt –β) Shift by β : [x(t -β)] Then Compress by a:[x(t- β) ⇒ x (αt-β)] OR Compress by α: [x(t)⇒ x(αt)] then Shift by $\frac{\beta}{\alpha}$:[x(αt) ⇒x{α (t- $\frac{\beta}{\alpha}$)} = x (αt –β)}]	**Derivative of impulse (doublet)**	$\frac{d}{dt}\delta(t) = \delta'(t) = \begin{cases} undefined, t = 0 \\ 0, otherwise \end{cases}$ $\delta'(\alpha t) = \frac{1}{\alpha\|\alpha\|}\delta'(t)$ $\int_{-\infty}^{\infty} x(t)\delta'(t-\lambda) dt = -x'(\lambda)$ $x(t)\delta'^{(t)} = x(0)\delta'^{(t)} - x'(0)\delta(t)$
Energy and power	Periodic signals have infinite energy hence power type signals.		