

Integral

Defining $u(t)$ and $v(t)$ as functions of t the following rules apply:

$$\bullet \int u + v dt = \int u dt + \int v dt$$

$$\bullet \int au dt = a \int u dt$$

$$\bullet \int a dt = at$$

$$\bullet \int \frac{d}{dt} u \cdot u^n dt = \frac{u^{n+1}}{n+1}$$

$$\bullet \int \frac{d}{dt} u \cdot e^u dt = e^u$$

$$\bullet \int \frac{d}{dt} u \cdot a^u dt = \frac{a^u}{\ln(a)}$$

$$\bullet \int \frac{\frac{d}{dt} u}{u} dt = \ln |u|$$

$$\bullet \int \frac{\frac{d}{dt} u}{1+u^2} dt = \tan^{-1}(u)$$

$$\bullet \int \frac{\frac{d}{dt} u}{\sqrt{1+u^2}} dt = \sin^{-1}(u)$$

Trigonometric

$$\bullet \int \frac{d}{dt} u \cdot \sin(u) dt = -\cos(u)$$

$$\bullet \int \frac{d}{dt} u \cdot \cos(u) dt = \sin(u)$$

$$\bullet \int \frac{d}{dt} u \cdot \tan(u) dt = -\ln|\cos(u)|$$

$$\bullet \int \frac{d}{dt} u \cdot \cot(u) dt = -\ln|\sin(u)|$$

$$\bullet \int \frac{d}{dt} u \cdot \sec(u)^2 dt = \tan(u)$$

$$\bullet \int \frac{d}{dt} u \cdot \csc(u)^2 dt = -\cot(u)$$

$$\bullet \int \frac{d}{dt} u \cdot \sec(u) dt = \ln|\sec(u) + \tan(u)|$$

$$\bullet \int \frac{d}{dt} u \cdot \csc(u) dt = \ln|\csc(u) + \cot(u)|$$

Leibniz Integral Rule: $\int u \cdot \frac{d}{dt} v dt = u \cdot v - \int v \cdot \frac{d}{dt} u$

Substitution Integral Rule: $\int u(t) dt = \int u(v(t)) \cdot \frac{d}{dt} v(t) dt$

Trigonometric substitutions:

$\sqrt{a^2 - t^2}$	$t = a \cdot \sin(x)$
$\sqrt{a^2 + t^2}$	$t = a \cdot \tan(x)$
$\sqrt{t^2 - a^2}$	$t = a \cdot \sec(x)$