Integral Examples

1. $\int 2t + t^3\, dt = t^2 + \frac{t^4}{4} + c\,, c \in \mathbb{R}$

2. $\int (2t)^5\, dt = \frac{1}{2}\int 2.(2t)^5\, dt = \frac{(2t)^6}{6} + c\,, c \in \mathbb{R}$

3. $\int e^{7t}\, dt = \frac{1}{7}\int 7.e^{7t}\, dt = \frac{e^{7t}}{7} + c\,, c \in \mathbb{R}$

4. $\int \frac{2t^5}{3+t^6} dt = \frac{1}{3}\int \frac{6t^5}{3+t^6} dt = ln\,|3 + t^6| + c\,, c \in \mathbb{R}$

5. $\int \frac{2t}{1+t^4} dt = tan^{-1}(t^2) + c\,, c \in \mathbb{R}$

6. $\int cos(3t)\, dt = \frac{1}{3}\int 3.cos(3t)\, dt = \frac{sin(3t)}{3} + c\,, c \in \mathbb{R}$

7. $\int cos(t)^3\, dt = \int cos(t).cos(t)^2\, dt = \int cos(t).(1 - sin(t))^2\, dt =$

$= \int cos(t) - cos(t).sin(t)^2\, dt = sin(t) - \frac{sin(t)^3}{3} + c\,, c \in \mathbb{R}$

8. $\int e^{2t}.t\, dt = \frac{e^{2t}}{2}.t - \int \frac{e^{2t}}{2} dt = \frac{e^{2t}}{2}.t - \frac{e^{2t}}{4} + c\,, c \in \mathbb{R}$ *(L. RULE)*

9. $\int \frac{1}{\sqrt{a^2+t^2}} dt = \int \frac{1}{\sqrt{(a.tan(x))^2+a^2}}.a\, sec(x)^2\, dx =$

$\int \frac{1}{a}\frac{1}{\sqrt{(tan(x))^2+1}}.a\, sec(x)^2\, dx$

$= \int \frac{sec(x)^2}{\sqrt{sec(x)^2}} dx = \int sec(x)\, dx = ln\,|sec(t) + tan(t)| + c\,, c \in \mathbb{R}$ *(T. SUBS.)*

10. $\int sec(t)^3\, dt = \int sec(t).sec(t)^2\, dt =$

$= sec(t).tan(t) - \int sec(t).tan(t)^2\, dt =$

$= sec(t).tan(t) - \int sec(t).(sec(t)^2 - 1)\, dt =$

$= sec(t).tan(t) - [\int sec(t)^3\, dt - \int sec(t)\, dt] =$

$= sec(t).tan(t) + ln\,|sec(t) + tan(t)| - \int sec(t)^3\, dt$

Thus $\int sec(t)^3\, dt = sec(t).tan(t) + ln\,|sec(t) + tan(t)| - \int sec(t)^3\, dt \Leftrightarrow$

$\Leftrightarrow 2\int sec(t)^3\, dt = sec(t).tan(t) + ln\,|sec(t) + tan(t)| \Leftrightarrow$

$\Leftrightarrow \int sec(t)^3\, dt = \frac{1}{2}[sec(t).tan(t) + ln\,|sec(t) + tan(t)\,|] + c\,, c \in \mathbb{R}$