

7. TRIGONOMETRIC FUNCTIONS

Synopsis :

1. Let $\theta \in \mathbb{R}$. Take an angle of measure θ radians in the standard position. Let $P(x, y)$ be a point on the terminal side of the angle θ such that $OP = r (> 0)$. Then

i) $\frac{y}{r}$ is called sine of θ and it is denoted by $\sin\theta$.

ii) $\frac{x}{r}$ is called cosine of θ and it is denoted by $\cos\theta$

iii) $\frac{y}{x}$ ($x \neq 0$) is called tangent of θ and it is denoted by $\tan\theta$.

iv) $\frac{x}{y}$ ($y \neq 0$) is called cotangent of θ and it is denoted by $\cot\theta$.

v) $\frac{r}{x}$ ($x \neq 0$) is called secant of θ and it is denoted by $\sec\theta$.

vi) $\frac{r}{y}$ ($y \neq 0$) is called cosecant of θ and it is denoted by $\operatorname{cosec}\theta$.

These six functions (ratios) are called trigonometric functions (ratios).

2. $\sin\theta \cdot \operatorname{cosec}\theta = 1$, $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

3. $\cos\theta \cdot \sec\theta = 1$, $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

4. $\tan\theta \cdot \cot\theta = 1$, $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

5. $\frac{\sin\theta}{\cos\theta} = \tan\theta$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$

6. $\sin^2\theta + \cos^2\theta = 1$,

$$\sin^2\theta = 1 - \cos^2\theta,$$

$$\cos^2\theta = 1 - \sin^2\theta$$

7. $1 + \tan^2\theta = \sec^2\theta$, $\tan^2\theta = \sec^2\theta - 1$, $\sec^2\theta - \tan^2\theta = 1$.

8. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, $\cot^2\theta = \operatorname{cosec}^2\theta - 1$,

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1.$$

9. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$.