FREQUENCY RESPONSE, BODE PLOT, RESONANCE

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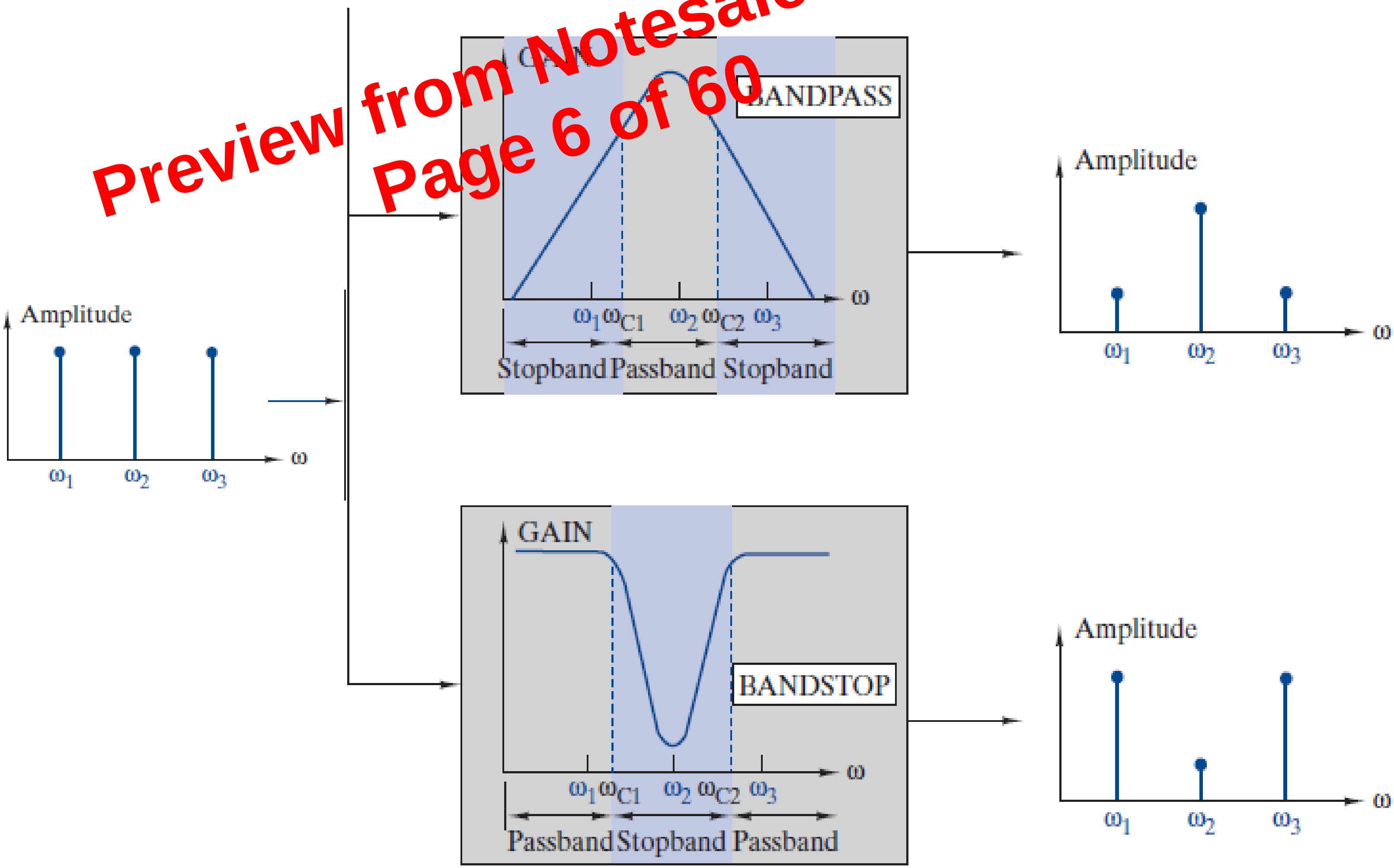
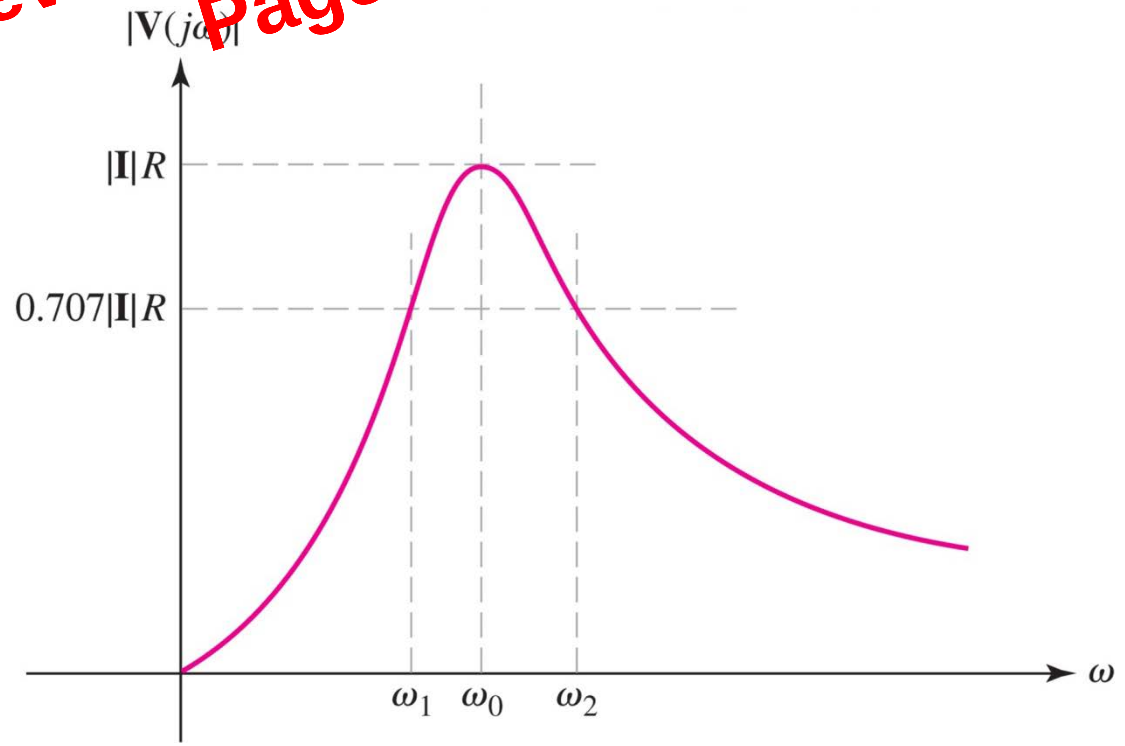
Basic Gain Responses Cont…



RESONANCE and the voltage response: | |L RLC



The Quality Factor Q: || EL RLC

$$Q = 2\pi \frac{[w_L(t) + w_C(t)]_{\max}}{P_R T}$$

P_R total average power absorbed by all the resistor

$$i(t) = \mathbf{I}_m \cos \omega_0 t \qquad v(t) = Ri(t) = R\mathbf{I}_m \cos \omega_0 t$$

$$w_C(t) = \frac{1}{2} C v^2 = \frac{\mathbf{I}_m^2 R^2 C}{2} \cos^2 \omega_0 t$$

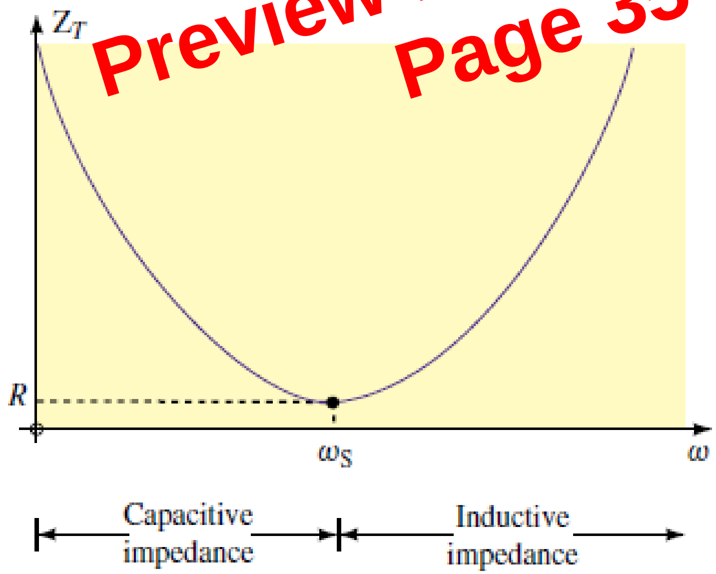
$$w_L(t) = \frac{1}{2} L i_L^2 = \frac{1}{2} L \left(\frac{1}{L} \int v \, dt \right)^2 = \frac{1}{2L} \left[\frac{R\mathbf{I}_m}{\omega_0} \sin \omega_0 t \right]^2$$

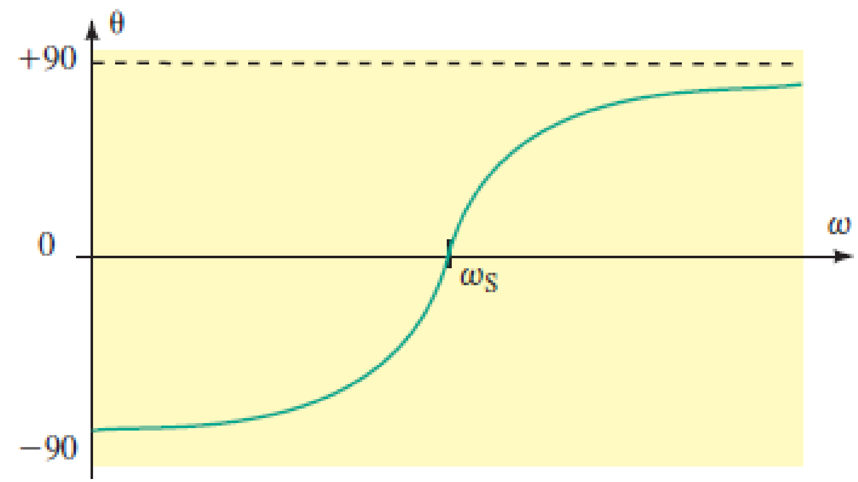
$$w_L(t) = \frac{\mathbf{I}_m^2 R^2 C}{2} \sin^2 \omega_0 t$$

The total *instantaneous* stored energy is therefore constant:

$$w(t) = w_L(t) + w_C(t) = \frac{\mathbf{I}_m^2 R^2 C}{2}$$

RESONANCE: Series RLC





Due to the changing impedance of the circuit, we conclude that if a constant amplitude voltage is applied to the series resonant circuit, the current and power of the circuit will not be constant at all frequencies.

RESONANCE: Series RLC

The bandwidth of the series resonant circuit is

$$BW = w_b = w_2 - w_1 = \frac{R}{L}$$

We define the Q (quality factor) of the circuit as;

$$Q = \frac{w_o L}{R} = \frac{1}{w_o RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

Using Q, we can write the bandwidth as;

$$BW = \frac{w_o}{Q}$$

These are all important relationships.

RESONANCE: Series RLC

An Observation

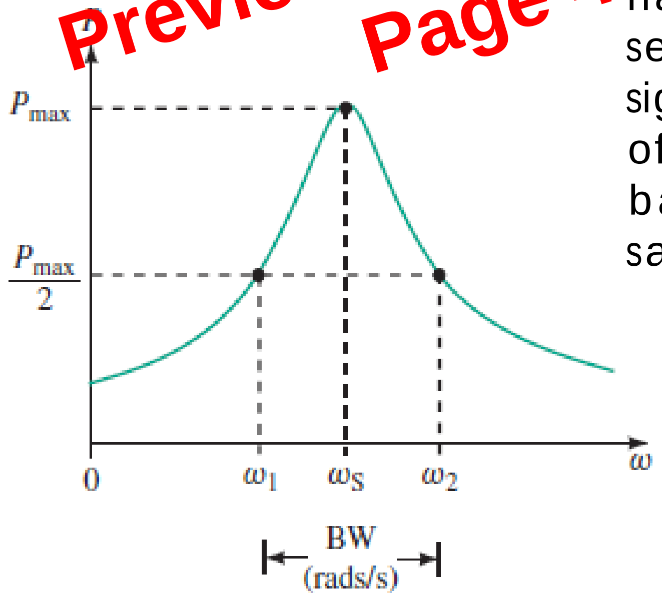
By using $Q = w_o L/R$ in the equations for w_1 and w_2 we have;

$$w_1 = w_o \left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

and

$$w_2 = w_o \left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

RESONANCE: Series RLC SELECTIVITY



Selectivity curve.

If the bandwidth of a circuit is kept very narrow, the circuit is said to have a high selectivity, since it is highly selective to signals occurring within a very narrow range of frequencies. On the other hand, if the bandwidth of a circuit is large, the circuit is said to have a low selectivity.

APPROXIMATE EXPRESSIONS

Q >= 10

$$\omega_d \approx \omega_0$$

$$\omega_{1,2} \approx \omega_0 \mp \frac{1}{2}B$$

$$\omega_0 \approx \frac{1}{2}(\omega_1 + \omega_2)$$

EXAMPLE 2

A series RLC resonant circuit has a resonant frequency admittance of $2x10^{-2}$ S(mohs). The Q of the circuit is 50, and the resonant frequency is 10,000 rad/sec. Calculate the values of R, L, and C. Find the half-power frequencies and the bandwidth.

<u>First,</u> R = 1/G = 1/(0.02) = 50 ohms.

<u>Second</u>, from $Q = \frac{w_o L}{R}$ **, we solve for L, knowing Q, R, and w_o to find L = 0.25 H.**

<u>Third,</u> we can use $C = \frac{Q}{w_o R} = \frac{50}{10{,}000x50} = 100 \ \mu F$