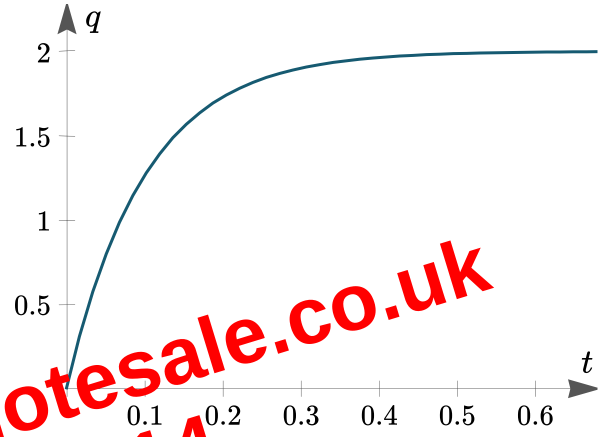

Graph of $q = 2\left(1 - e^{-10t}\right)$, solution of a differential equation.

As $t \to \infty$, $q \to 2$ C

Now,

$$V_C = \frac{1}{C} \int i\, dt$$

$$= \frac{1}{C} q$$

$$= \frac{1}{0.02} 2\left(1 - e^{-10t}\right)$$

$$= 100\left(1 - e^{10t}\right)$$

For comparison, here is the solution of the DE using **variables separable**:

$$\frac{dq}{dt} = 10(2 - q)$$

$$\frac{dq}{2 - q} = 10\, dt$$

$$-\ln|2 - q| = 10t + K$$

(We could continue and get the same expression as above.)

Since $t = 0$, $q = 0$, we have $K = -\ln 2$.

Then we use the integration formula (found in our standard integral table):

$$\int e^{mt}\cos nt\,dt = \frac{e^{mt}}{m^2+n^2}(m\cos nt + n\sin nt)$$

We obtain:

$$e^{25t}q = 8.5\int e^{25t}\cos 150t\,dt$$

$$= 8.5\frac{e^{25t}}{23125}(25\cos 150t + 150e^{25t}\sin 150t)$$

$$= 0.0092e^{25t}\cos 150t + 0.055e^{25t}\sin 150t + K$$

Dividing throughout by e^{25t} gives:

$$q = 0.0092\cos 150t + 0.055\sin 150t + Ke^{-25t}$$

We now need to find K:

$q(0) = -0.05$ means $K = -0.05 - 0.0092 = -0.0592$

So this gives us:

$$q = 0.0092\cos 150t + 0.055\sin 150t - 0.0592e^{-25t}$$

Method Using Scientific Notebook

We set up the differential equation and the initial conditions in a **matrix** (not a table) as follows:

$$\frac{dq}{dt} + 25q = 8.5\cos 150t$$

$$q(0) = -0.05$$

Choosing **Solve ODE - Exact** from the **Compute** menu gives:

Exact solution is:

$$q(t) = 0.0092\cos 150t + 0.055\sin 150t - 0.059e^{-25t}$$

The graph for $q(t)$ is as follows: