## MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms are to the base $e$ unless otherwise specified.
(2) The set of all $x$ such that $a \leq x \leq b$ is denoted by $[a, b]$.

1. If $f(g(x)) = 5$ and $f(x) = x + 3$ for all real $x$, then $g(x) =$

(A) $x - 3$ (B) $3 - x$ (C) $\frac{5}{x+3}$ (D) 2 (E) 8

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2 $\lim_{x \to 0} \frac{\tan x}{\cos x} =$

(A) $-\infty$ (B) $-1$ (C) 0 (D) 1 (E) $+\infty$

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3. $\int_0^{\log 4} e^{2x} dx =$

(A) $\frac{15}{2}$ (B) 8 (C) $\frac{7}{2}$ (D) $\frac{\log 16}{2} - 1$ (E) $\log 4 - \frac{1}{2}$

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4. Let $A - B$ denote $\{x \in A: x \notin B\}$. If $(A - B) \cup B = A$, which of the following must be true?

(A) $B$ is empty
(B) $A \subseteq B$
(C) $B \subseteq A$
(D) $(B - A) \cup A = B$
(E) None of the above

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5. If $f(x) = |x| + 3x^2$ for all real $x$, then $f'(-1)$ is

(A) $-7$ (B) $-5$ (C) 5 (D) 7 (E) nonexistent

6. For what value of $b$ is the value of $\int_b^{b+1} (x^2 + x)\,dx$ a minimum?

(A) 0 (B) $-1$ (C) $-2$ (D) $-3$ (E) $-4$

7. In how many of the eight standard octants of $xyz$-space does the graph of $z = e^{x+y}$ appear?

(A) One (B) Two (C) Three (D) Four (E) Eight

8. Suppose that the function $f$ is defined on an interval by the formula $f(x) = \sqrt{\tan^2 x - 1}$. If $f$ is continuous, which of the following intervals could be its domain?

(A) $\left(\frac{3\pi}{4}, \pi\right)$

(B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(D) $\left(-\frac{\pi}{4}, 0\right)$

(E) $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$



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9. $\int_0^1 \frac{x}{2 - x^2}\, dx =$

(A) $-\frac{1}{2}$ (B) $\frac{5}{3}$ (C) $\frac{\log 2 - e}{2}$ (D) $-\frac{\log 2}{2}$ (E) $\frac{\log 2}{2}$

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10. If $f''(x) = f'(x)$ for all real $x$, and if $f(0) = 0$ and $f'(0) = -1$, then $f(x) =$

(A) $1 - e^x$ (B) $e^x - 1$ (C) $e^{-x} - 1$ (D) $e^{-x}$ (E) $-e^x$

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11. If $\phi(x, y, z) = x^2 + 2xy + xz^{\frac{3}{2}}$, which of the following partial derivatives are identically zero?

I. $\frac{\partial^2 \phi}{\partial y^2}$

II. $\frac{\partial^2 \phi}{\partial x \partial y}$

III. $\frac{\partial^2 \phi}{\partial z \partial y}$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

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41. If $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, then the set of all vectors $X$ for which $AX = X$ is

(A) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a = 0 \text{ and } b \text{ is arbitrary} \right\}$

(B) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a \text{ is arbitrary and } b = 0 \right\}$

(C) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a = -b \text{ and } b \text{ is arbitrary} \right\}$

(D) $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

(E) the empty set

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42. What is the greatest value of $b$ for which any real-valued function $f$ that satisfies the following properties must also satisfy $f(1) < 5$?

(i) $f$ is infinitely differentiable on the real numbers;
(ii) $f(0) = 1$, $f'(0) = 1$, and $f''(0) = 2$; and
(iii) $|f'''(x)| < b$ for all $x$ in $[0, 1]$.

(A) 1 (B) 2 (C) 6 (D) 12 (E) 24

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56. For a subset $S$ of a topological space $X$, let $\text{cl}(S)$ denote the closure of $S$ in $X$, and let $S' = \{x: x \in \text{cl}(S - \{x\})\}$ denote the derived set of $S$. If $A$ and $B$ are subsets of $X$, which of the following statements are true?

I. $(A \cup B)' = A' \cup B'$
II. $(A \cap B)' = A' \cap B'$
III. If $A'$ is empty, then $A$ is closed in $X$.
IV If $A$ is open in $X$, then $A'$ is not empty.

(A) I and II only
(B) I and III only
(C) II and IV only
(D) I, II, and III only
(E) I, II, III, and IV

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57. Consider the following procedure for determining whether a given name appears in an alphabetized list of $n$ names.

Step 1. Choose the name at the middle of the list (if $n = 2k$, choose the $k$th name); if that is the given name, you are done; if the list is only one name long, you are done. If you are not done, go to Step 2.

Step 2. If the given name comes alphabetically before the name at the middle of the list, apply Step 1 to the first half of the list; otherwise, apply Step 1 to the second half of the list.

If $n$ is very large, the maximum number of steps required by this procedure is close to

(A) $n$
(B) $n^2$
(C) $\log_2 n$
(D) $n \log_2 n$
(E) $n^2 \log_2 n$

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