The Wavelet Transform

An Introduction

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the Different frequency contents that are presents in the signal.

In frequency domain representation of the signal there is a graph between frequency and amplitude .In this frequency Amplitude graph The frequency of the signal is taken at the x axis as independent variable whereas the Amplitude of the signal is taken at y Axis. The frequency domain representation of the signal tells us that in a given signal what different frequency components are present and what their respective amplitudes are. But again frequency domain representation gives no idea that at which time these frequency components are present. In some applications frequency domain representation is more important. Or we can say that frequency domain representation gives more information about any signal (for example in any audio music signal).

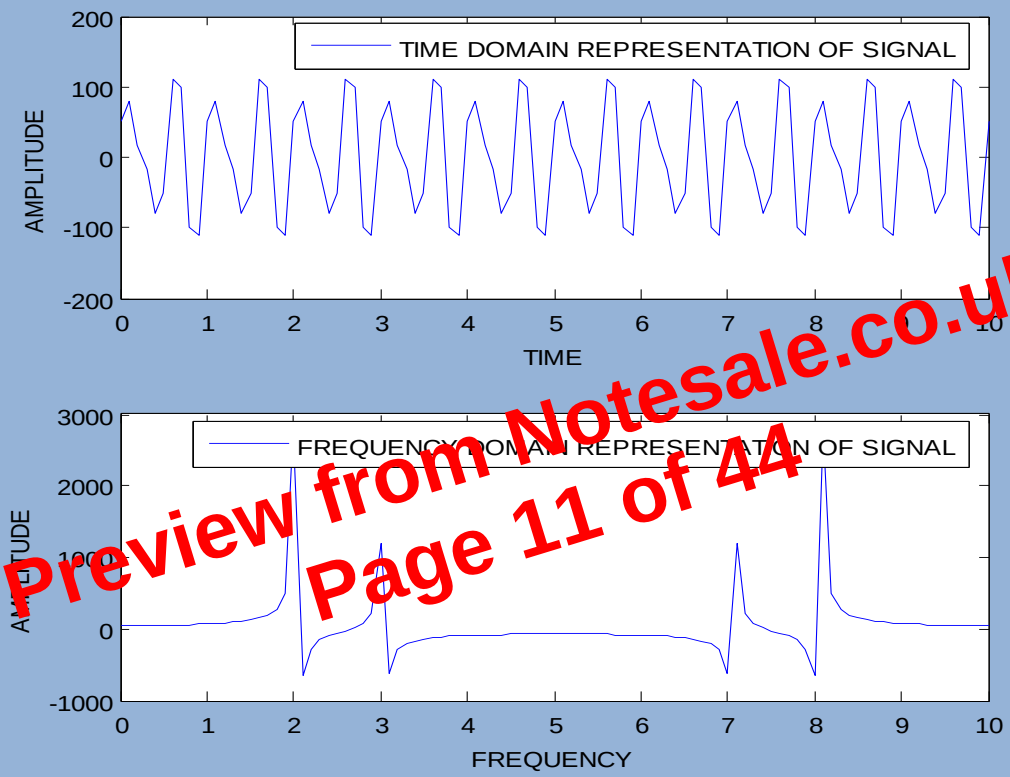
Suppose

x= 100*sin(2*pi*2*t)+50* cos (2*pi*3*t)

is a time Domain representation of a given signal and

y=fft(x)

is frequency doamin representaion of signal x



As it is clear from fig (1) that time domain representation of signal gives no idea about the frequency components of signal it simply shows that how the amplitude of the signal is varying with respect to time whereas frequency domain representation of the signal shows the different frequency components presents in a signal and their respective amplitudes but it gives no idea that at which time instants these frequency components presents.

each frequency In the other word we can say that equation 1 take a frequency for example f_1 and search it from –infinity to + infinity over time if it find the f_1 frequency components it simply adds the magnitude of all f1 frequency components.

Again take an another frequency for example f_2 and search it from – infinity to + infinity over time if it find the f_2 frequency components it simply adds the magnitude of all f2 frequency components

Again repeat the same process with f3, f_4, f 5…… and so on

No matter in time axis where these frequency components exits from – infinity to + infinity it will effect the result of integration in the same way

For every frequency Fourier transform check that whether this particular frequency component present or not present in time from minus infinite to plus infinite. And if present then how many times this particular frequency component presents and what is the amplitude of this particular frequency component and then simply add that particular frequency component and calculate the amplitude of any particular frequency component.

Again take a second frequency component and check that in time from minus infinite to plus infinite how many times this

particular frequency component exists and what is amplitude of this particular frequency component and then simply adds them. In this way the Fourier transform calculated the amplitude of every frequency components presents in a given signal and draw a graph between frequency and amplitude

Again there are certain disadvantage of frequency domain representation of the signal first disadvantage is that it gives no idea about time .The frequency transform of any signal simply tells us that in any given signal what spectral components are present and what are their respective amplitudes but it gives no idea that in time axis where these frequency components exists

So again the D.F.T. prove its suitability for the signals which are stationary in nature but this transform is not suitable for non stationary signal

By stationary signal we simply means the signal in which the frequency does not change with respect to time or we can say that all frequency components exits for all the time

By non stationary signal we simply mean the signal in which frequency changes with respect to the time. or in which all the frequency components does not exist for all the time interval .but some frequencies are exits for some particular

signal cannot be considered stationary(because the signal is stationary only for the short time interval)

Again at the same time we can't get good time and frequency resolution either we get good time resolution or good frequency resolution

The small window size is suitable for high frequencies whereas

Large window size is suitable for low frequencies

But the problem with STFT is that the window size remains same for the all analysis we can't choose different -different window Size for the analysis of different frequency components.

As shown in fig that for all frequencies the size of window is same

Once window size is chosen then we cannot change the size of window .And any single window size cannot suitable for different frequency components

Again in S.T.F.T. it is very tough task to choose the size of window

Again with wavelet transform we have freedom to design our own wavelet hence we can define our own wavelet by defining Two functions

[1]Wavelet function:

[2]Scale function:

[1] Wavelet function: wavelet function capture the details (high frequencies) present in any signal and the integration of wavelet function should be zero or the mean value of wavelet function should be zero

$$\int \Psi(x).d(x) = 0$$

[2] Scale function: Scale function capture the low frequencies information (approximate) presents in any signal. the integration of scale function should be one it means its average value is one.

$$\int Ø(x).d(x) = 1$$