Concepts	Description
	Valuation and Analysis : Bonds with Embedded Options
Embedded options	**Embedded option :** allow issuer to (1) manage interest rate risk amd/or (2) issue the bonds at attractive coupon rate **1. Callable bonds :** give issuer the option to call back the bond → investor is short the call option - **European style :** option can only be exercised on 1 single day, immidiately after the lockout period - **American style :** option can be exercised at anytime after the lockout period - **Bermudan style :** option can be exercised at fixed dates after the lockout period **2. Putable bonds :** allow investor to sell the bond back to issuer prior to maturity → investor is long the put option **3. Estate put :** allow the heirs of an investor to sel the bond back to the issuer upon death of the investor **4. Sinking fund bonds :** require issuer to set aside funds periodically to retire the bond
Relationship between value of callable/putable bonds, straight bonds and embedded option	$V_{Callable\ bond} = V_{straight} - V_{Call\ option}$ $V_{putable\ bond} = V_{straight} + V_{put\ option}$
Value a bond with embedded options using binomial tree framework	**1. Valuing callable bond :** Value at any node where the bond is callable = Call price or Computed value if bond is not called, whichever is lower **2. Valuing putable bond :** Value at any node where the bond is putable = Put price or Computed value if bond is not put, whichever is higher
Effect of volatility on value of callable/putable bond	Option values are positively related to volatility of their underlying - ↑ interest rate volatility → value of call/put option increase → value of callable bond decreases, value of putable bond increases
Impact of change in level of interest rates on value of callable / putable bond	- ↓ interest rate → call option limits upside potential → value of callable bond rise less rapidly than the value of equivalent straight bond - ↑ interest rate → put option limits downside potential → value of putable bond fall less rapidly than the value of equivalent straight bond
Impact of change in shape of yield curve on value of callable / putable bond	- ↑ interest rate → ↓ probability of call option being in the money → ↓ value of call option for an upward sloping yield curve - ↑ interest rate → ↑ probability of put option being in the money → ↑ value of put option for an upward sloping yield curve
Option-adjusted spreads	Option-adjusted spread (OAS) : constant spread added to all one-period rates in the binominal tree, so that the calculated value = Market price of risky bond Impact of volatility on OAS: - ↑ volatility → ↑ Calculated value of call option ; No impact on straight bond → ↓ Calculated of callable bond → Closer to market price → ↓ OAS - ↑ volatility → ↑ Calculated value of put option ; No impact on straight bond → ↑ Calculated of putable bond → Further to market price → ↑ OAS
Effective duration / Effective convexity	**Effecttive duration :** Measure price sensitivity to interest rate changes of bond with embedded option, assumes that CF does change due to change in interest rate **Effective convexity :** Explain the change in price that is not explained by duration of bond with embedded option, assume that CF does change due to change in interest rate $Effective\ duration = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$ $Effective\ convexity = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{BV_0 \times \Delta y^2}$ $\Delta y = Change\ in\ required\ yield$ $BV_{-\Delta y} = Estimated\ price\ if\ yield\ decrease\ by\ \Delta y$ $BV_{+\Delta y} = Estimated\ price\ if\ yield\ increase\ by\ \Delta y$ $BV_0 = initial\ observe\ bond\ price$ Method for calculating estimated price if yield change by Δy - Step 1 : Given assumption about benchmark interest rates, interest volatility, and any calls/puts → calculate OAS for the bond, based on current market price and binominal model - Step 2 : Impose a parallel shift in the benchmark yield curve by +Δy - Step 3 : Build a new binominal interest rate tree, using the new yield curve - Step 4 : Add the OAS (Step 1) to the interest rate tree to get a "modified" tree - Step 5 : Calculate the new estimated price, if yield change by +Δy, using modified interest rate tree - Step 6 : Repeat Step 2-5, using parallel rate shift of -Δy
Compare effective duration of callable/putable vs straight bond	Effective duration (Callable) ≤ effective duration (straight) Effective duration (Putable) ≤ effective duration (straight) Effective duration (zero-coupon) ≈ maturity of the bond Effective duration of fixed-rate bond < Maturity of the bond Effective duration of floater ≈ years to next reset of interest rate
One-sided duration	**One-sided durations :** durations that apply only when interest rates rise /or fall (applied for bonds with embedded options) - Call option is at-the-money : One-sided down duration (price change of callable bond when interes rates fall) < One-sided up-duration (price change of callable bond when interest rates rise) - Put option is at-the-money : One-sided up duration (price change of putable bond when interest rate rise) < One-sided down-duration (price change of putable bond when interest rate fall)
Key rate duration (Partial duration)	**Key rate duration :** capture the interest rate sensitivity of a bond to changes in yields of specific benchmark maturities → identify the interest rate risk from change in the shape of yield curve Process of computing key rate duration : similar to the process of computing effective duration , except that only 1 specific key eate is shifted before the price impact is measured **Observe about key rates:** 1. option-free bond is trading at par → bond's maturity-matched rate is the only rate that affects the bond's value. Maturity key rate duration = Effective duration; all other rate durations = 0 2. option-free bond is not trading @ par → bond's maturity-matched is still the most important rate 3. Bond with low/zero coupon rate → might have negative key rate duration for horizons other than its maturity 4. Callable bonds with low coupon rate → unlikely to be called → maturity-matched rate is the most critical rate 5. Higher coupon bonds → more likely to be called → time-to-exercise rate will tend to dominate time-to-maturity rate 6. Putable bonds with high coupon rate → unlikely to be put → maturity-matched rate is the most critical rate 7. Lower coupon bonds → more likely to be put → time-to-exercise rate will tend to dominate time-to-maturity rate