Black model for valuation of Interest rate option	Interest rate option : option to enter a FRA in the future. Value of a call option on an (M x N) FRA can be calculated as $C_0 = AP \times e^{-r\times(Actutal/365)} \times [FRA_{M\times N} \times N(d_1) - X \times N(d_2)] \times Notional\ principal$ $AP = accrual\ period = Actual/365$ Equivalencies: - Long FRA = Long interest rate call + Short interest rate put (exercise rate = current FRA rate) - Short FRA = Short interest rate call + Long interest rate put (exercise rate = current FRA rate) - Interest rate cap = series of interest rate call with different maturities and same exercise price - Interest rate floor = series of interest rate put with different maturities and same exercise price - Payer swap = long cap + short floor (exercise rate on cap = exercise rate on floor) - Exercise rate on floor = exercise rate on cap = market swap fixed rate → value on cap = value on floor
Black model for valuation of Swaption	**Swaption :** option that gives the holder the right to enter into interest rate swap **Payer swaption :** fixed-rate payer (receive float) **Receiver swaption :** fixed-rate receiver (pay float) Swaption = option on series of CF (annuity) @ each settlement date of the underlying swap that equal to difference between exercise rate on swaption and market swap fixed rate $PAY = AP \times PVA \times [SFR \times N(d_1) - X \times N(d_2)] \times Notional\ principal$ $REC = AP \times PVA \times [X \times N(-d_2) - SFR \times N(-d_1)] \times Notional\ principal$ In which: $PAY = payer\ swaption$ $REC = receiver\ swaption$ $AP = 1/Number\ of\ settlement\ periodsper\ year\ of\ the\ underlying\ swap$ $SFR = current\ marke\ swap\ fixed\ rate$ $d_1 = \frac{ln(SFR/X) + (\sigma^2/2) \times T}{\sigma \times \sqrt{T}}$ $d_2 = d_1 - \sigma \times \sqrt{T}$ Equivalencies - Receiver swap = Long receiver swaption + Short payer swaption (same exercise rates) - Payer swap = Short receiver swaption + Long payer swaption (same exercise rates) - If exercise rate is set so that values of payer swaption = values of receiver swaption → exercise rate - market swap fixed rate - Long callable bond = option-free bond + short receiver swaption
Option Greeks	Greeks : sensitivity that capture the relationship between each input and the option price - Inputs : asset price, exercise price, asset price volatility, time to expiration, risk-free rate - Greeks : + Delta : relationship between changes in asset price and changes in option price + Gamma : capture the curvature of option value vs. stock price relationship → the rate of change in delta + Vega : measure sensitivity of option price to changes in volatility of return on underlying asset + Rho : measure the sensitivity of option price to changes in risk-free rate + Theta : measure the sensitivity of option price to the passage of time
Option Greeks - Delta	Delta : relationship between changes in asset price and changes in option price - Call option delta : positive - ↑ Underlying asset price → ↑ Call option value - Put option delta : negative - ↑ Underlying asset price → ↓ Put option value Delta calculation : $Delta_C = e^{-\delta\times T} \times N(d_1)\ \rightarrow Out\ of\ the\ money:\ Delta_C\ moves\ closer\ to\ 0\ ; In\ the\ money:\ Delta_C\ moves\ closer\ to\ e^{-\delta\times T}$ $Delta_P = e^{-\delta\times T} \times N(-d_1) \rightarrow Out\ of\ the\ money:\ Delta_P moves\ closer\ to\ 0\ ; In\ the\ money:\ Delta_P moves\ closer\ to\ -e^{-\delta\times T}$ Relationship between changes Call / Put option value vs Changes in asset price $\Delta C = Delta_C \times \Delta S$ $\Delta P = Delta_P \times \Delta S$
Option Greeks - Gamma	Gamma : Capture the curvation of option value vs. stock price relationship → the rate of change in delta Long position in calls and puts : positive gamma - Short option → lower gamma - Long option → increase gamma Gamma is highest for at-the-money options Gamma is low for deep-in-the-money or deep-out-of-money $\Delta C = Delta_C \times \Delta S + {}^1/_2 \times Gamma \times \Delta S^2$ $\Delta P = Delta_P \times \Delta S + {}^1/_2 \times Gamma \times \Delta S^2$
Option Greeks - Vega	Vega : measure the sensitivity of the option price to changes in volatility of returns on underlying asset Higher volatility → Increase value f call / put option → positive vega for both call / put
Option Greeks - Rho	Rho : measure the sensitivity of the option price to change in risk-free rate (*) Price of European call / put option does not change mch if use different inpts for risk-free rate
Option Greeks - Theta	Theta : sensitivity of option price to passage of time Call / Put option approach maturity → decrease spculative vaulue → call / put value decrease (except for deep-in-the-money put options, that might increase value as time passes)
Delta Hedge	Delta-neutral portfolio (delta-neutral hedge) : long stock position + short call (or long put) option position → value of portfolio does not change as stock price changes $Number\ of\ short\ call\ needed\ for\ delta\ hedge = \frac{number\ of\ shares\ hedged}{delta\ of\ call\ option}$ $Number\ of\ long\ put\ needed\ for\ delta\ hedge = \frac{number\ of\ shares\ hedged}{delta\ of\ put\ option}$
Gamma risk	Gamma risk : risk that stock price might suddenly jump, leaving delta-hedged portfolio unhedged