has presented the matrix $A \in C^{N\times N}$ is normed if and only if for all vectors $x \in C^N$

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$$\left\|A^{n+m}\right\|_2 \leq \sqrt{\left\|A^{2n}x\right\|_2 \left\|A^{2m}\right\|_2},$$

for all $n, m = 0, 1, \ldots$ where $\| \ \|_2$ be the Euclidean norm on C^N. The Lexicographic order is a total in C compatible with addition of complex numbers and multiplication by positive real and it is characterized by its positive cone

$$H = \{\alpha + i\beta : \alpha > 0 \text{ or if } \alpha = 0, \beta > 0\}.$$

The compatibility with addition is $H + H \subseteq H$ which compatibility with multiplication by positive real, is $\lambda H \subseteq H$ for $\lambda > 0$. The order being total is $H \cup -H = C \setminus \{0\}$. The lexicographic order is not Archimedian and apart from rotation if the positive cone is the only total order in C compatible with these addition and multiplication operations. The difference between hermitian and general normal matrices is that they can have as eigen values arbitrary complex number C of course is not an ordered field, but it turns out the simple fact that C can be totally ordered as a vector space over the reals if enough to find useful information on spectra of normal matrices by using hermitian matrices as an inspiration was given by [23].

H-Unitary Matrix

A complex matrix that are unitary with respect to indefinite inner product induced by an invertible hermitian matrix H is said to be H-unitary matrix.

Lorentz Matrix

The real matrices that are orthogonal with respect to indefinite inner product induced by an invertible real symmetric matrices are said to be Lorentz matrices.

Let $M_n = M_n(F)$ be the algebra of $n \times n$ square matrices with entries in the field F = C, the complex numbers, or F = R the real numbers, and if $H \in M_n$ is an invertible hermitian matrix, a matrix $A \times M_n$ is said to be H-unitary if $H^* HA = H$. The authors of [24] and [25] have been presented applications of H-unitary valued functions in engineering and interpolation and for an exposition from the point of view of numerical method were studied by [26]. Several canonical forms of H-unitary matrices and demonstrate some of its applications was established by [27].

Conjugate Normal Matrices

Let $M_n(C)$ be the set of complex $n \times n$ matrices and a matrix $A \in M_n(C)$ is called conjugate normal matrix if

$$AA^* = \overline{A^* A}. \tag{6}$$

It plays an important role in the theory of unitary congruences as conventional normal matrices do in the theory of unitary similarities. We can easily verify that matrix $A \in M_n(C)$ is conjugate normal if and only if the corresponding matrix $\hat{A}$ is normal in the conventional sense, where

$$\hat{A} = \begin{bmatrix} 0 & A \\ \overline{A} & 0 \end{bmatrix}.$$

One of the most useful criteria that $A \in M_n(C)$ is normal if and only if the hermitian adjoint A^* can be represented as a polynomial of A as $A^* = f(A)$.

Let the spectrum of A is $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, then desired polynomial f can be obtained by Lagranges interpolation

$$f(\lambda_i) = \overline{\lambda_i}, i = 1, 2, \ldots, n-1.$$

The degree of polynomial is at most n-1, and it coefficients are in general complex. The author of [28] used this criterion to show the following result:

Result

A matrix $A \in M_n(C)$ is conjugate normal if and only if the transpose A^T can be represented in the form

$$A^T = g(A_k)A$$

where g is a polynomial with real coefficients.

Condiagonalization

A matrix $A \in M_n(C)$ is called condiagonalizable if $A_R = A\overline{A}$ or $A_L = \overline{A}A$ is diagonalizable by a similar transformation or we can say that matrix $A \in M_n(C)$ is condiagonalizable if there exists a non-singular $S \in M_n(C)$ such that $S^{-1}A\overline{S}$ is diagonal.

The author of [29] has given a description of condiagonalizable matrices that would be more elementary then the use of the Canonical Jordan like form. He proved that any condiagonalizable matrix can be brought by a consimilar transformation to a special block diagonal form with the diagonal blocks of order 1 or 2.

Let λ be a simple eigen value of a normal matrix A, then its condition number attains the minimal possible value 1. In most general case where matrix A have multiple eigen values, a suitable characterization of ideal condition can be obtained from the Bauer-Fike theorem as below:

B.*Bauer-Fike Theorem*

Let $M_n(C)$ be the set of nxn complex matrices and a matrix $A \in M_n(C)$ be a diagonalizable matrix with eigen value decomposition

$$A = P \wedge P^{-1} \tag{7}$$