Q.14 $a_n = \frac{(\ln n)^2}{n}$

Sol. $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(\ln n)^2}{n}$ ($\frac{\infty}{\infty}$ form)

$= \lim_{n\to\infty} \frac{2 \ln n \cdot \frac{1}{n}}{1}$ (using L'Hospital rule)

$= \lim_{n\to\infty} \frac{2 \ln n}{n}$ ($\frac{\infty}{\infty}$ form)

$= \lim_{n\to\infty} \frac{2 \cdot \frac{1}{n}}{1} = \lim_{n\to\infty} \frac{2}{n} = 0$

$\boxed{\lim_{n\to\infty} a_n = 0}$ ∴ seq converges to '0'.

Q15. $a_n = \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)$ Rationalize

Sol: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right) = \lim_{n\to\infty} \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)\frac{(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})}$

$= \lim_{n\to\infty} \frac{\sqrt{n}\,(\cancel{n}+1-\cancel{n})}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$ $\left(\frac{\infty}{\infty}\right)$

$= \lim_{n\to\infty} \frac{\cancel{\sqrt{n}}}{\left[\sqrt{\frac{n+1}{n}}+1\right]\cancel{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+1}{n}}+1}$

$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{2}$

$\boxed{\lim_{n\to\infty} a_n = \frac{1}{2}}$ ∴ Seq Converges to $\frac{1}{2}$.

Q16 $a_n = \frac{5^n + (-1)^n}{5^{n+1} + (-1)^{n+1}} = \frac{5^n + (-1)^n}{5 \cdot 5^n + (-1)^n(-1)}$

Sol

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5^n + (-1)^n}{5 \cdot 5^n + (-1)^n(-1)} = \lim_{n\to\infty} \frac{\cancel{5^n}\left(1 + \frac{(-1)^n}{5^n}\right)}{\cancel{5^n}\left(5 - \frac{(-1)^n}{5^n}\right)}$

$= \lim_{n\to\infty} \frac{1 + \frac{(-1)^n}{5^n}}{5 - \frac{(-1)^n}{5^n}} = \frac{1}{5}$

$\because (-1)^n = \begin{cases} -1, & \text{for } n \text{ is odd} \\ 1, & \text{for } n \text{ is even} \end{cases}$

$\boxed{\lim_{n\to\infty} a_n = \frac{1}{5}}$ ∴ Seq $\{a_n\}$ converges to $\frac{1}{5}$