

$$\frac{du}{dx} = e^{4x}(\cos x) + \sin x(4e^{4x})$$

$$= e^{4x}(\cos x + 4\sin x)$$

$$v = x \cos 2x$$

$$c = x; d = \cos 2x$$

$$\frac{dc}{dx} = 1; \frac{dd}{dx} = -2\sin 2x$$

$$\frac{dv}{dx} = x(-2\sin 2x) + \cos 2x$$

$$= \cos 2x - 2x\sin 2x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\begin{aligned} & (x \cos 2x)(e^{4x}(\cos x + 4\sin x)) - \\ & = \frac{e^{4x} \sin x (\cos 2x - 2x \sin 2x)}{(x \cos 2x)^2} \end{aligned}$$

$$6. y = \frac{x^4}{(x+1)^2}$$

$$u = x^4, \frac{du}{dx} = 4x^3$$

$$v = (x+1)^2$$

$$w = x+1; v = w^2$$

$$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$$

$$= 2w \times 1$$

$$= 2(x+1)$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(x+1)^2(4x^3) - (x^4)(2)(x+1)}{(x+1)^4}$$

$$= \frac{2x^4 + 4x^3}{(x+1)^3}$$

EXERCISE 3

$$1. y = \frac{e^{4x}}{x^3 \cosh 3x}$$

take $\ln$ of both sides

$$\ln y = \ln e^{4x} - \ln x^3 - \ln \cosh 3x$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{e^{4x}} \cdot 4e^{4x} - \frac{1}{x^3} \cdot 3x^2 - \frac{1}{\cosh 3x} \cdot 3 \sinh 3x$$

$$\frac{dy}{dx} = \frac{e^{4x}}{x^3 \cosh 3x} \left[4 - \frac{3}{x} - 3 \tanh 3x \right]$$

$$2. y = \frac{(3x+1)\cos 2x}{e^{2x}}$$

take $\ln$ of both sides

$$\ln y = \ln(3x+1) + \ln \cos 2x - \ln e^{2x}$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{3x+1} \cdot 3 + \frac{1}{\cos 2x} \cdot (-2\sin 2x) - \frac{1}{e^{2x}} \cdot 2e^{2x}$$

$$\frac{dy}{dx} = \frac{(3x+1)\cos 2x}{e^{2x}} \left[\frac{3}{3x+1} - 2\tan 2x - 2 \right]$$

$$3. y = x^5 \sin 2x \cos 4x$$

take $\ln$ of both sides

$$\ln y = \ln x^5 + \ln \sin 2x + \ln \cos 4x$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

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$$\frac{1}{x^5} \cdot 5x^4 + \frac{1}{\sin 2x} \cdot 2\cos 2x + \frac{1}{\cos 4x} \cdot -4\sin 4x$$

$$\frac{dy}{dx}$$

$$= x^5 \sin 2x \cos 4x \left[\frac{5}{x} + 2\cot 2x - 4\tan 4x \right]$$

$$4. y = \frac{(x^3-1)\sin 5x}{x^6}$$

take $\ln$ of both sides

$$\ln y = \ln (x^3 - 1) + \ln \sin 5x - \ln x^6$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} =$$

$$\frac{1}{x^3-1} \cdot 3x^2 + \frac{1}{\sin 5x} \cdot 5\cos 5x - \frac{1}{x^6} \cdot 6x^5$$

$$\frac{dy}{dx} = \frac{(x^3-1)\sin 5x}{x^6} \left[\frac{3x^2}{x^3-1} + 5\cot 5x - \frac{6}{x} \right]$$

$$5. y = \frac{\sin 2x \cos 3x}{\cos 4x}$$

take $\ln$ of both sides

$$\ln y = \ln \sin 2x + \ln \cos 3x - \ln \cos 4x$$

differentiating wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2\cos 2x}{\sin 2x} - \frac{3\sin 3x}{\cos 3x} + \frac{4\sin 4x}{\cos 4x}$$

$$\frac{dy}{dx}$$

$$= \frac{\sin 2x \cos 3x}{\cos 4x} [2\cot 2x - 3\tan 3x + 4\tan 4x]$$

EXERCISE 4.

$$1. y = \cos(7x + 2)$$

$$\text{let } u = 7x + 2; y = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\sin u \times 7$$

$$= -7\sin(7x + 2)$$

$$2. y = (4x - 5)^6$$

$$\text{let } u = 4x - 5; y = u^6$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 6u^5 \times 4$$

$$= 24u^5$$

$$= 24(4x - 5)^6$$

$$3. y = e^{3-x}$$

$$\text{let } u = 3 - x; y = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= e^u \times -1$$

$$= -e^u$$

$$= -e^{3-x}$$

$$4. y = \sin 2x$$

$$\text{let } u = 2x; y = \sin u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 2$$

$$= 2\cos u$$

$$= 2\cos 2x$$

$$5. y = \cos(x^2)$$

$$\text{let } u = x^2; y = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\sin u \times 2x$$

$$= -2x\sin(x^2)$$

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$$y' = 3x^2$$

$$y'' = 6x$$

$$5. y = 2x^3 + 3x^2 - 12x + 20$$

$$y' = 6x^2 + 6x - 12$$

$$y'' = 12x + 6$$

EXERCISE 6

$$1. x^3 + y^3 - 3xy = 8$$

$$3x^2 + 3y^2y' - 3[xy' + y] = 0$$

$$3x^2 + 3y^2y' - 3xy' - 3y = 0$$

$$y'[3y^2 - 3x] = 3y - 3x^2$$

$$y' = \frac{3y - 3x^2}{3y^2 - 3x} = \frac{y - x^2}{y^2 - x}$$

$$2. x^2 + y^3 - 4x + 4y = 26$$

$$2x + 3y^2y' - 4 + 4y' = 0$$

$$2 + 6yy'' = 0$$

$$y'' = -\frac{2}{6y}$$

$$= -\frac{1}{3y}$$

$$3. x^3 + y^3 + 4xy^2 = 5$$

$$3x^2 + 3y^2y' + 4[x \cdot 2yy' + y^2] = 0$$

$$3y^2y' + 8xyy' = -4y^2 - 3x^2$$

$$y'[3y^2 + 8xy] = -4y^2 - 3x^2$$

$$y' = \frac{-4y^2 - 3x^2}{3y^2 + 8xy}$$

$$4. x^2 + y^2 - 5xy^3 + 9 = 0$$

$$2x + 2yy' - 5[x \cdot 3y^2y' + y^3] = 0$$

$$y'[2y - 15xy^2] = 5y^3 - 2x$$

$$y' = \frac{5y^3 - 2x}{2y - 15xy^2}$$

CHAPTER 5

Exercise 1

$$1. y = x^2 - x$$

$$\text{Area} = \int_0^1 y dx$$

$$= \int_0^1 (x^2 - x) dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_0^1$$

$$= \left[\frac{1^3}{3} - \frac{1^2}{2} \right] - \left[\frac{0^3}{3} - \frac{0^2}{2} \right]$$

$$= \frac{1}{3} - \frac{1}{2}$$

$$= \frac{2-3}{6}$$

$$= -\frac{1}{6}$$

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