$\frac{du}{dx} = e^{4x}(\cos x) + sinx(4e^{4x})$

$= e^{4x}(cosx + 4sinx)$

$v = xcos2x$

$c = x; d = cos2x$

$\frac{dc}{dx} = 1; \frac{dd}{dx} = -2sin2x$

$\frac{dv}{dx} = x(-2sin2x) + cos2x$

$= cos2x - 2xsin2x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(xcos2x)\left(e^{4x}(cosx + 4sinx)\right) - e^{4x}sinx(cos2x - 2xsin2x)}{(xcos2x)^2}$

6. $y = \frac{x^4}{(x+1)^2}$

$u = x^4; \frac{du}{dx} = 4x^3$

$v = (x + 1)^2$

$w = x + 1; v = w^2$

$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$

$= 2w \times 1$

$= 2(x + 1)$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(x + 1)^2(4x^3) - (x^4)(2)(x + 1)}{(x + 1)^4}$

$= \frac{2x^4 + 4x^3}{(x + 1)^3}$

EXERCISE 3

1. $y = \frac{e^{4x}}{x^3cosh3x}$

take In of both sides

$In\, y = In\, e^{4x} - In\, x^3 - In\, cosh3x$

differentiating wrt x

$\frac{1}{y}.\frac{dy}{dx} =$

$\frac{1}{e^{4x}}.4e^{4x} - \frac{1}{x^3}.3x^2 - \frac{1}{cosh3x}.3sinh3x$

$\frac{dy}{dx} = \frac{e^{4x}}{x^3cosh3x}[4 - \frac{3}{x} - 3tanh3x]$

2. $y = \frac{(3x+1)cos2x}{e^{2x}}$

take In of both sides

$In\, y = In\,(3x + 1) + In\, cos2x - In\, e^{2x}$

differentiating wrt x

$\frac{1}{y}.\frac{dy}{dx} =$

$\frac{1}{3x+1}.3 + \frac{1}{cos2x}.(-2sin2x) - \frac{1}{e^{2x}}.2e^{2x}$

$\frac{dy}{dx} = \frac{(3x+1)cos2x}{e^{2x}}[\frac{3}{3x+1} - 2tan2x - 2]$

3. $y = x^5sin2xcos4x$

take In of both sides

$In\, y = In\, x^5 + In\, sin2x + In\, cos4x$

differentiating wrt x

$\frac{1}{y}.\frac{dy}{dx} =$

$\frac{1}{x^5}.5x^4 + \frac{1}{sin2x}.2cos2x + \frac{1}{cos4x}.-4sin4x$

$\frac{dy}{dx}$

$= x^5 sin2xcos4x[\frac{5}{x} + 2cot2x - 4tan4x]$

4. $y = \frac{(x^3-1)sin5x}{x^6}$

take In of both sides

$In\ y = In\ (x^3 - 1) + In\ sin5x - In\ x^6$

differentiating wrt x

$\frac{1}{y}.\frac{dy}{dx} =$

$\frac{1}{x^3-1}.3x^2 + \frac{1}{sin5x}.5cos5x - \frac{1}{x^6}.6x^5$

$\frac{dy}{dx} = \frac{(x^3-1)sin5x}{x^6}[\frac{3x^2}{x^3-1} + 5cot5x - \frac{6}{x}]$

5. $y = \frac{sin2xcos3x}{cos4x}$

take In of both sides

$In\ y = In\ sin2x + In\ cos3x - In\ cos4x$

differentiating wrt x

$\frac{1}{y}.\frac{dy}{dx} = \frac{2cos2x}{sin2x} - \frac{3sin3x}{cos3x} + \frac{4sin4x}{\cos 4x}$

$\frac{dy}{dx}$

$= \frac{sin2xcos3x}{cos4x}[2cot2x - 3tan3x + 4tan4x]$

EXERCISE 4.

1. $y = \cos(7x + 2)$

$let\ u = 7x + 2; y = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -sinu \times 7$

$= -7\sin(7x + 2)$

2. $y = (4x - 5)^6$

$let\ u = 4x - 5; y = u^6$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 6u^5 \times 4$

$= 24u^5$

$= 24(4x - 5)^6$

3. $y = e^{3-x}$

$let\ u = 3 - x; y = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times -1$

$= -e^u$

$= -e^{(3-x)}$

4. $y = \sin 2x$

$let\ u = 2x; y = \sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= cosu \times 2$

$= 2cosu$

$= 2cos2x$

5. $y = \cos(x^2)$

$let\ u = x^2; y = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -sinu \times 2x$

$= -2x\sin(x^2)$

$y' = 3x^2$

$y'' = 6x$

5. $y = 2x^3 + 3x^2 - 12x + 20$

$y' = 6x^2 + 6x - 12$

$y'' = 12x + 6$

EXERCISE 6

1. $x^3 + y^3 - 3xy = 8$

$3x^2 + 3y^2y' - 3[xy' + y] = 0$

$3x^2 + 3y^2y' - 3xy' - 3y = 0$

$y'[3y^2 - 3x] = 3y - 3x^2$

$$y' = \frac{3y - 3x^2}{3y^2 - 3x} = \frac{y - x^2}{y^2 - x}$$

2. $x^2 + y^3 - 4x + 4y = 26$

$2x + 3y^2y' - 4 + 4y' = 0$

$2 + 6yy'' = 0$

$y'' = -\frac{2}{6y}$

$= -\frac{1}{3y}$

3. $x^3 + y^3 + 4xy^2 = 5$

$3x^2 + 3y^2y' + 4[x.2yy' + y^2] = 0$

$3y^2y' + 8xyy' = -4y^2 - 3x^2$

$y'[3y^2 + 8xy] = -4y^2 - 3x^2$

$$y' = \frac{-4y^2 - 3x^2}{3y^2 + 8xy}$$

4. $x^2 + y^2 - 5xy^3 + 9 = 0$

$2x + 2yy' - 5[x.3y^2y' + y^3] = 0$

$y'[2y - 15xy^2] = 5y^3 - 2x$

$$y' = \frac{5y^3 - 2x}{2y - 15xy^2}$$

CHAPTER 5

Exercise 1

1. $y = x^2 - x$

$$Area = \int_0^1 y\,dx$$

$$= \int_0^1 (x^2 - x)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \left[\frac{1^3}{3} - \frac{1^2}{2}\right] - \left[\frac{0^3}{3} - \frac{0^2}{2}\right]$$

$$= \frac{1}{3} - \frac{1}{2}$$

$$= \frac{2-3}{6}$$

$$= -\frac{1}{6}$$