Continue...

- Statistical or Empirical Probability (VON MISES): If an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials becomes indefinitely large, is called the probability of happening of the event, it is being assumed that the limit is finite and unique. Mathematically, if for n trials an event A happens m times, then

$$P(A) = \lim_{n \to \infty} \frac{m}{n}.$$

- J. E. Kerrich conducted a coin tossing experiment with 10 sets of 1000 tosses ech during his confinement in World war-II. The number of heads found were: 502, 511, 529, 504, 476, 507, 520, 504, 529. The probability of getting a head in tossing a coin once is computed using the above definition, $5,079/10,000 = 0.5079 \approx 0.5$.

Bonferroni's Inequality

- Given n events, $A_1, A_2, \ldots, A_n$, we have

$$\sum_{i=1}^{n} P(A_i) \geq P(\bigcup_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - \sum_{1\leq i\leq j\leq n} P(A_i \bigcap A_j).$$

Proof: This can be proved by the method of mathematical induction. Check that, the result is true for $n = 3$.

$$\begin{aligned} P(\bigcup_{i=1}^{3}) &= \sum_{i=1}^{3} P(A_i) - \sum_{1\leq i<j\leq 3} P(A_i \bigcap A_j) + P(\bigcap_{i=1}^{3} A_i) \\ &\geq \sum_{i=1}^{3} P(A_i) - \sum_{1\leq i<j\leq 3} P(A_i \bigcap A_j). \end{aligned}$$

Let the result is true for $n = k$. To prove the result for $n = k + 1$.