$$\nabla.\mathrm{v} = \frac{\partial x^5 y^3}{\partial x} + \frac{\partial y^2 z^4}{\partial y} - \frac{\partial x z^2}{\partial z}$$

$\nabla.\mathrm{v} = 5x^4y^3 + 2yz^4 - 2xz$

$\nabla.\mathrm{v}\ (1,2,3) = 5(1)^4(2)^3 + 2(2)(3)^4 - 2(1)(3)$

$\nabla.\mathrm{v}\ (1,2,3) = 40 + 324 - 6$

$$\nabla.\mathrm{v}\ (1,2,3) = 358$$

The Curl

The curl is the product of two vectors delivering the result in vector form. The difference between the curl and divergent is that curl delivers the product of two vectors in vector form while divergent delivers in scalar form. This is accomplished by applying cross product.

$\nabla \mathrm{X}\ \mathrm{v} = (\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k)\ \mathrm{X}\ (v_1\mathrm{i}+v_2\mathrm{j}+v_3\mathrm{k})$

$$\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_1 & V_2 & V_3 \end{vmatrix}$$

$= \mathrm{i}\ (\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z}) - \mathrm{j}\ (\frac{\partial V_3}{\partial x} - \frac{\partial V_1}{\partial z}) + \mathrm{k}\ (\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y})$

Using the above pattern, we can solve for curl of any given vector.

Example

Find the curl of $x^5y^3\mathrm{i} + y^2z^4\mathrm{j} + xz^2\mathrm{k}$ at (1,2,3)

$$\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^5y^3 & y^2z^4 & xz^2 \end{vmatrix}$$

$= \mathrm{i}\ (\frac{\partial xz^2}{\partial y} - \frac{\partial y^2z^4}{\partial z}) - \mathrm{j}\ (\frac{\partial xz^2}{\partial x} - \frac{\partial x^5y^3}{\partial z}) + \mathrm{k}\ (\frac{\partial y^2z^4}{\partial x} - \frac{\partial x^5y^3}{\partial y})$