Exercise 1A

Question 1:

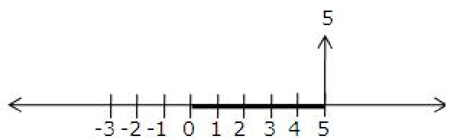
The numbers of the form $\frac{p}{q}$, where p and q are integers and q ≠ 0 are known as rational numbers.

Ten examples of rational numbers are:

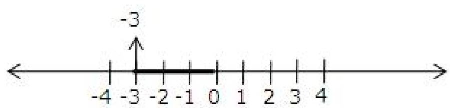
$\frac{2}{3}, \frac{4}{5}, \frac{7}{9}, \frac{8}{11}, \frac{15}{23}, \frac{23}{27}, \frac{25}{31}, \frac{26}{32}, 1, \frac{12}{5}$

Question 2:

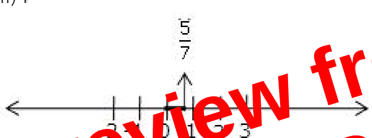
(i) 5



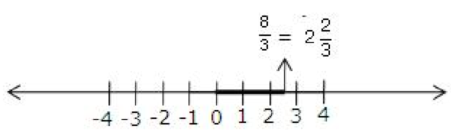
(ii) -3



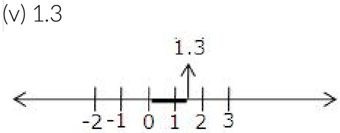
(iii) $\frac{5}{7}$



(iv) $\frac{8}{3} = 2\frac{2}{3}$



(v) 1.3



Subtracting (i) from (ii) we get;

9x = 12

⇒ x = $\frac{12}{9} = \frac{4}{3}$

Hence, $1.\bar{3}$ = $\frac{4}{3}$

(iii) Let x = $0.\overline{34}$

i.e x = 0.3434 .... (i)

⇒ 100x = 34.3434 .... (ii)

Subtracting (i) from (ii), we get

99x = 34

⇒ x = $\frac{34}{99}$

Hence, $0.\overline{34}$ = $\frac{33}{99}$

(iv) Let x = $3.\overline{14}$

i.e x = 3.1414 .... (i)

⇒ 100x = 314.1414 .... (ii)

Subtracting (i) from (ii), we get

99x = 311

⇒ x = $\frac{311}{99}$

Hence, $3.\overline{14}$ = $\frac{311}{99}$

(v) Let x = $0.\overline{324}$

i.e. x = 0.324324 ....(i)

⇒ 1000x = 324.324324....(ii)

Subtracting (i) from (ii), we get

999x = 324

⇒ x = $\frac{324}{999} = \frac{12}{37}$

Hence, $0.\overline{324}$ = $\frac{12}{37}$

(vi) Let x = $0.1\bar{7}$

i.e. x = 0.177 .... (i)

⇒ 10x = 1.777 .... (ii)

and 100x = 17.777.... (iii)

Subtracting (ii) from (iii), we get

90x = 16

⇒ x = $\frac{16}{90} = \frac{8}{45}$

Hence, $0.1\bar{7}$ = $\frac{8}{45}$

(vii) Let x = $0.5\bar{4}$

i.e. x = 0.544 .... (i)

⇒ 10 x = 5.44 .... (ii)

and 100x = 54.44 ....(iii)

Subtracting (ii) from (iii), we get

90x = 49

⇒ x = $\frac{49}{90}$

Hence, $0.5\bar{4}$ = $\frac{49}{90}$

(vii) Let x = Let x = $0.1\overline{63}$

i.e. x = 0.16363 .... (i)

⇒ 10x = 1.6363 .... (ii)

and 1000 x = 163.6363 .... (iii)

Subtracting (ii) from (iii), we get

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then
$\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other,
as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}=\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$=\frac{\left(4+\sqrt{5}\right)^2}{(4)^2-\left(\sqrt{5}\right)^2}$$
$$=\frac{(4)^2+2(4)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2}{16-5}$$
$$\frac{4+\sqrt{5}}{4-\sqrt{5}}=\frac{16+8\sqrt{5}+5}{11}=\frac{21+8\sqrt{5}}{11}\ldots\ldots(1)$$

Now consider the denominator of the second

term on the left hand side:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$=\frac{\left(4-\sqrt{5}\right)^2}{(4)^2-\left(\sqrt{5}\right)^2}$$
$$=\frac{(4)^2-2(4)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2}{16-5}$$
$$\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{16-8\sqrt{5}+5}{11}=\frac{21-8\sqrt{5}}{11}\ldots\ldots(2)$$

Adding equations (1) and (2), we have,

$$\therefore\ \frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{21+8\sqrt{5}}{11}+\frac{21-8\sqrt{5}}{11}$$
$$=\frac{21+8\sqrt{5}+21-8\sqrt{5}}{11}=\frac{42}{11}.$$

Question 16:

Given, $x=\left(4-\sqrt{15}\right)$

Then,

$$\left(x+\frac{1}{x}\right)=\left(4-\sqrt{15}+\frac{1}{4-\sqrt{15}}\right)$$
$$=\left(4-\sqrt{15}+\frac{1}{4-\sqrt{15}}\times\frac{4+\sqrt{15}}{4+\sqrt{15}}\right)\quad\text{[rationalisation]}$$
$$=\left(4-\sqrt{15}+\frac{4+\sqrt{15}}{(4)^2-\left(\sqrt{15}\right)^2}\right)$$
$$=\left(4-\sqrt{15}+\frac{4+\sqrt{15}}{16-15}\right)$$
$$=\left(4-\sqrt{15}+\frac{4+\sqrt{15}}{1}\right)$$
$$=4-\sqrt{15}+4+\sqrt{15}=8.$$

Question 17: