Laplace Properties

1.) Linearity

$$L[f_1(t) + f_2(t)] = L[f_1(t)] + L[f_2(t)] = F_1(s) + F_2(s)$$

$$L[af_1(t)] = aL[f_1(t)] = aF_1(s)$$

2.) Time Scaling

Let $g(t) = f(at)$ for $a > 0$

$$G(s) \coloneqq L[g(t)] = \frac{1}{a}F\left(\frac{s}{a}\right)$$

3.) Exponential Scaling

Let $g(t) = e^{at}f(t)$ for some scalar a

$$G(s) = F(s - a)$$

4.) Time Delay

Let $g(t) = \begin{cases} f(t-T), & t \geq T \\ 0, & \leq t < T \end{cases}$

$$L[g(t)] = e^{-sT}F(s)$$

5.) Derivative

Let $g(t) = \frac{d}{dt}f(t)$

$$L[g(t)] = sF(s) - f(0)$$

Let $g(t) = \frac{d^2}{dt^2}f(t)$

$$L[g(t)] = s^2F(s) - sf(0) - f'(0)$$

Let $g(t) = \frac{d^3}{dt^3}f(t)$

$$L[g(t)] = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

6.) Integral

Let $g(t) = \int_0^t f(T)\,dT$

$$L[g(t)] = \frac{1}{s}F(s)$$

7.) Multiplication by t

Let $g(t) = tf(t)$

$$L[g(t)] = -\frac{d}{ds}F(s)$$

8.) Convolution

$$f * g = \int_0^t f(T)\,g(t-T)dT$$

$$f * g = g * f$$

Now let $h(t) = f(t) * g(t)$

$$L[h(t)] = F(s)G(s)$$