

lengths.

$\therefore$ BDEF is a parallelogram.

(ii) We have proved that BDEF is a parallelogram.

Similarly, DCEF is a parallelogram and DEAF is also a parallelogram.

Now, parallelogram BDEF and parallelogram DCEF are on the same base EF and between the same parallels BC and EF.

$$\therefore \text{ar}(\parallel^{\text{gm}} \text{BDEF}) = \text{ar}(\parallel^{\text{gm}} \text{DCEF})$$

$$\Rightarrow \frac{1}{2} \text{ar}(\parallel^{\text{gm}} \text{BDEF}) = \frac{1}{2} \text{ar}(\parallel^{\text{gm}} \text{DCEF})$$

$$\Rightarrow \text{ar}(\triangle \text{BDF}) = \text{ar}(\triangle \text{CDE}) \dots (1)$$

[Diagonal of a parallelogram divides it into two triangles of equal area]

$$\text{Similarly, ar}(\triangle \text{CDE}) = \text{ar}(\triangle \text{DEF}) \dots (2)$$

$$\text{and ar}(\triangle \text{AEF}) = \text{ar}(\triangle \text{DEF}) \dots (3)$$

From (1), (2) and (3), we have

$$\text{ar}(\triangle \text{AEF}) = \text{ar}(\triangle \text{BDF}) = \text{ar}(\triangle \text{DEF}) = \text{ar}(\triangle \text{CDE})$$

$$\text{Thus, ar}(\triangle \text{ABC}) = \text{ar}(\triangle \text{AEF}) + \text{ar}(\triangle \text{BDF}) + \text{ar}(\triangle \text{DEF}) + \text{ar}(\triangle \text{CDE}) = 4 \text{ ar}(\triangle \text{DEF})$$

$$\Rightarrow \text{ar}(\triangle \text{DEF}) = \frac{1}{4} \text{ar}(\triangle \text{ABC})$$

$$\text{(iii) We have, ar}(\parallel^{\text{gm}} \text{BDEF}) = \text{ar}(\triangle \text{BDF}) + \text{ar}(\triangle \text{DEF})$$

$$= \text{ar}(\triangle \text{DEF}) + \text{ar}(\triangle \text{DEF}) [\because \text{ar}(\triangle \text{DEF}) = \text{ar}(\triangle \text{BDF})]$$

$$2 \text{ar}(\triangle \text{DEF}) = 2 \left[\frac{1}{4} \text{ar}(\triangle \text{ABC}) \right]$$

$$= \frac{1}{2} \text{ar}(\triangle \text{ABC})$$

$$\text{Thus, ar}(\parallel^{\text{gm}} \text{BDEF}) = \frac{1}{2} \text{ar}(\triangle \text{ABC})$$

Ex 9.3 Class 9 Maths Question 6.

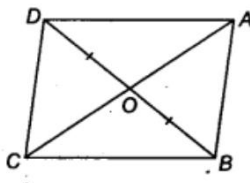
In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If

AO = CO, then show that

$$\text{(i) ar}(\triangle \text{DOC}) = \text{ar}(\triangle \text{AOB})$$

$$\text{(ii) ar}(\triangle \text{DCB}) = \text{ar}(\triangle \text{ACB})$$

(iii) DA || CB or ABCD is a parallelogram

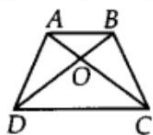


Ex 9.3 Class 9 Maths Question 15.

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that $ar(\triangle AOD) = ar(\triangle BOC)$. Prove that ABCD is a trapezium.

Solution:

We have a quadrilateral ABCD and its diagonals AC and BD intersect at O such that $ar(\triangle AOD) = ar(\triangle BOC)$ [Given]



Adding $ar(\triangle AOB)$ to both sides, we have

$$ar(\triangle AOD) + ar(\triangle AOB) = ar(\triangle BOC) + ar(\triangle AOB)$$

$$\Rightarrow ar(\triangle ABD) = ar(\triangle ABC)$$

Also, they are on the same base AB.

Since, the triangles are on the same base and having equal area.

$\therefore$ They must lie between the same parallels.

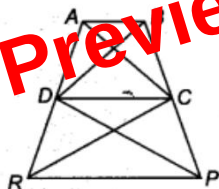
$$\therefore AB \parallel DC$$

Now, ABCD is a quadrilateral having a pair of opposite sides parallel.

So, ABCD is a trapezium.

Ex 9.3 Class 9 Maths Question 16.

In figure $ar(\triangle DRC) = ar(\triangle DPC)$ and $ar(\triangle BDP) = ar(\triangle ARC)$. Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Solution:

We have, $ar(\triangle DRC) = ar(\triangle DPC)$ [Given]

And they are on the same base DC.

$\therefore \triangle DRC$ and $\triangle DPC$ must lie between the same parallels.

So, $DC \parallel RP$ i.e. a pair of opposite sides of quadrilateral DCPR is parallel.

$\therefore$ Quadrilateral DCPR is a trapezium.

Again, we have

$$ar(\triangle BDP) = ar(\triangle ARC) \text{ [Given] } \dots(1)$$

$$\text{Also, } ar(\triangle DPC) = ar(\triangle DRC) \text{ [Given] } \dots(2)$$

Subtracting (2) from (1), we get

$$ar(\triangle BDP) - ar(\triangle DPC) = ar(\triangle ARC) - ar(\triangle DRC)$$

$$\Rightarrow ar(\triangle BDC) = ar(\triangle ADC)$$

And they are on the same base DC.

$\therefore$ ABDC and AADC must lie between the same parallels.