Task 2.3

Question 1a:

$$\cos^2\theta - sin^2\theta = 2cos^2\theta - 1$$

$$\text{LHS} = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta - \sin^2\theta = 1 \quad SO \quad sin^2\theta = 1 - cos^2\theta$$

$$LHS = cos^2\theta - (1 - cos^2\theta)$$

$$= cos^2\theta - 1 + cos^2\theta$$

$$= 2cos^2\theta - 1 = RHS \quad SO \quad cos^2\theta - sin^2\theta = 2cos^2\theta - 1$$

Question 1b:

$$\frac{1 + \text{cotA}}{1 + \text{tanA}} = \text{cotA}$$

$$1 + \text{cotA} = \text{cosecA}$$

$$1 + \text{tanA} = \text{secA}$$

$$\text{LHS} = \frac{1 + \text{cotA}}{1 + \text{tanA}} = \frac{\text{cosecA}}{\text{secA}}$$

SO AS $\text{secA} = \frac{1}{\text{cosA}}$ & $\text{cosecA} = \frac{1}{\text{sinA}}$

$$\text{LHS} = \frac{\text{cosecA}}{\text{secA}} = \frac{\frac{1}{\text{cosA}}}{\frac{1}{\text{sinA}}}$$

$$\frac{\text{sinA}}{\text{cosA}} = \text{cotA} \quad \text{SO} \quad \frac{1 + \text{cotA}}{1 + \text{tanA}} = \text{cotA}$$

Question 1c:

$$\frac{\text{sechA} - \text{cosechA}}{\text{tanhA} - \text{cothA}} = \text{coshA} - \text{sinhA}$$

$$\text{sechA} = \frac{2}{e^{A} + e^{-A}}$$

$$\text{cosechA} = \frac{2}{e^{A} - e^{-A}}$$

$$\text{tanhA} = \frac{e^{A} - e^{-A}}{e^{A} + e^{-A}}$$

$$\text{cothA} = \frac{e^{A} + e^{-A}}{e^{A} - e^{-A}}$$

$$\text{coshA} = \frac{e^{A} + e^{-A}}{2}$$

$$\text{sinhA} = \frac{e^{A} - e^{-A}}{2}$$

LHS:

$$\frac{\frac{2}{e^{A} + e^{-A}} - \frac{2}{e^{A} - e^{-A}}}{\frac{e^{A} - e^{-A}}{e^{A} + e^{-A}} - \frac{e^{A} + e^{-A}}{e^{A} - e^{-A}}}$$

$$= \frac{2\left(\frac{1}{e^{A} + e^{-A}} - \frac{1}{e^{A} - e^{-A}}\right)}{\left(\frac{(e^{A} - e^{-A})^{2} - (e^{A} + e^{-A})^{2}}{(e^{A} + e^{-A}) \times (e^{A} - e^{-A})}\right)}$$

$$= 2\frac{\left(\frac{(e^{A} - e^{-A}) - (e^{A} + e^{-A})}{(e^{A} - e^{-A} \times (e^{A} - e^{-A})}\right)}{\left(\frac{(e^{A} - e^{-A})^{2} - (e^{A} + e^{-A})^{2}}{(e^{A} + e^{-A}) \times (e^{A} - e^{-A})}\right)}$$

$$= \frac{\frac{-4e^{-A}}{(e^{A} + e^{-A}) \times (e^{A} - e^{-A})}}{\frac{(e^{A} - e^{-A})^{2} - (e^{A} + e^{-A})^{2}}{(e^{A} + e^{-A}) \times (e^{A} - e^{-A})}}$$

$$= \frac{-4e^{-A}}{(e^{A} + e^{-A}) \times (e^{A} - e^{-A})} \times \frac{(e^{A} + e^{-A}) \times (e^{A} - e^{-A})}{(e^{A} - e^{-A})^{2} \times (e^{A} + e^{-A})^{2}}$$

$$= \frac{-4e^{-A}}{(e^{A} - e^{-A})^{2} - (e^{A} + e^{-A})^{2}}$$