• A set must be well defined, meaning that its elements can be described and listed without ambiguity. For example: { 1, 3, 5 } and { letters of the English alphabet }.

• Two sets are called equal if they have exactly the same elements. - The order is irrelevant. - Any repetition of an element is ignored.

• If a is an element of a set S, we write $a \in S$.

• If b is not an element of a set S, we write $b \notin S$

EXERCISE 1

a) Specify the set A by listing its elements, where A = { whole numbers less than 100 divisible by 16 }.

b) Specify the set B by giving a written description of its elements, where B = { 0, 1, 4, 9, 16, 25 }.

c) Does the following sentence specify a set? C = { whole numbers close to 50 }.

Finite and infinite sets

All the sets we have seen so far have been **finite** sets, meaning that we can list all their elements.

Here are two more examples:

{ whole numbers between 2000 and 2005 } = { 2001, 2002, 2003, 2004 }

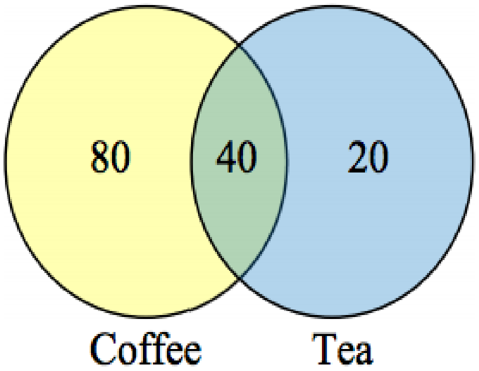
{ whole numbers between 2000 and 3000 } = { 2001, 2002, 2003,…, 2999 }

The three dots '…' in the second example stand for the other 995 numbers in the set. We could have listed them all, but to save space we have used dots instead. This notation can only be used if it is completely clear what it means, as in this situation.

A set can also be infinite – all that matters is that it is well defined. Here are two examples of infinite sets:

{ even whole numbers } = { 0, 2, 4, 6, 8, 10, …}

{ whole numbers greater than 2000 } = { 2001, 2002, 2003, 2004, …}



EXAMPLE

A survey asks: "Which online services have you used in the last month?"

- Twitter
- Facebook
- Have used both

The results show 40% of those surveyed have used Twitter, 70% have used Facebook, and 20% have used both. How many people have used neither Twitter or Facebook?

Answers

Let T be the set of all people who have used Twitter, and F be the set of all people who have used Facebook. Notice that while the cardinality of F is 70% and the cardinality of T is 40%, the cardinality of $F \cup T$ is not simply 70% + 40%, since that would count those who use both services twice. To find the cardinality of $F \cup T$, we can add the cardinality of F and the cardinality of T, then subtract those in intersection that we've counted twice. In symbols,

$n(F \cup T) = n(F) + n(T) - n(F \cap T)$

$n(F \cup T) = 70\% + 40\% - 20\% = 90\%$

Now, to find how many people have not used either service, we're looking for the cardinality of $(F \cup T)^c$. Since the universal set contains 100% of people and the cardinality of $F \cup T = 90\%$, the cardinality of $(F \cup T)^c$ must be the other 10%.

NOTE

The above example shows that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A^c) = n(U) - n(A)$

O_1 Trichotomy Postulate: For any real numbers a and b, exactly one of the following is true, a<b, a=b,a>b.

O_2 Transitive postulate: For any real numbers a, b and c, if a<b and b<c, then a<c

O_3 Addition postulate: For any real numbers a, band c, if a<b, then a + c < b + c

O_4 Multiplication postulate: For real numbers a, b and c, if a<b and 0<c, then ca < cb.

EXAMPLE

State one order postulate that justifies each of the following statements:

a) If -1<2 and 2<7, then -1<7

b) If -3<1, then 0<4

c) If 6<10 then 3<5.

SOLUTION

a) Transitive postulate O_2

b) Addition postulate O_3, by hypothesis -3<1, by O_3 $-3 + 3 < 1 + 3$ or 0<4

c) Multiplication postulate O_4, by hypothesis 6<10, by O_4 $6 \cdot \frac{1}{2} < 10 \cdot \frac{1}{2}$ or 3<5.

Theorem

For any real numbers a, b and c the following statements are true

a<b if and only if (iff) –a>-b

a>b iff –a<-b

If a<b and c<0, then ac>bc

If a<b and 0<c, then a/c < b/c

If a<b and c<d, then a + c < b + d

If a>0, c>0, a<b and c<d, then ac < bd

EXAMPLE

Prove that a<b iff –a>-b

SOLUTION

By hypothesis $a<b$, and by the addition postulate O_3, $a + \{(-a) + (-b)\} < b + \{(-a) + (-b)\}$,

i.e, $-b < -a$ or $-a > -b$.

EXAMPLE

Solve the following inequalities

(a) $5x \leq x - 3$ (b) $7 < 2x + 3 \leq 13$

SOLUTION

(a) $5x - x \leq (x - 3) - x$, thus $4x \leq 3$ or $x \leq ¾$. Solution set = $\{x \mid x \leq 3/4\}$.

(b) $7 - 3 < (2x + 3) - 3 \leq 13 - 3$, so $4 < 2x \leq 10$, dividing through by two we get: $2 < x \leq 5$.

Thus the solution set is: $\{ 2 < x \leq 5\}$.

The distance between two points on the number line is always indicated by a positive real number. The absolute value of a real number a is denoted by $|a|$ and is the distance of a from the origin

Absolute value = $|\pm a| = +a$, e.g $|-5| = 5$ and $|0| = 0$. If $|y| = b$, then $y = b$ or $y = -b$.

Let a and b be any two real numbers such that $a<b$. Each of the following sets is called an interval and is abbreviated as shown below.