$\sqrt{2}$ irrationality

$\alpha, \beta \in \mathbb{N}$	**given**
$a \stackrel{\text{def}}{=} \dfrac{\alpha}{\text{HCF}(\alpha,\beta)} \quad (a \in \mathbb{N})$ $b \stackrel{\text{def}}{=} \dfrac{\beta}{\text{HCF}(\alpha,\beta)} \quad (b \in \mathbb{N})$	**definitions**

(1)	$\text{HCF}\left[\dfrac{\alpha}{\text{HCF}(\alpha,\beta)}, \dfrac{\beta}{\text{HCF}(\alpha,\beta)}\right] = 1$	**T** **{Theorem}**	
(2)	$\text{HCF}(a,b) = 1 \ (a,b \in \mathbb{N})$	**T** {definition of a and b}	
(3)	$\text{CF}(a,b) = 2$	**F** {definition of HCF}	
(4)	a has factor 2 ^ b has factor 2	**F** {definition of CF}	
(5)	a has factor 2 $\Leftrightarrow$ $a = 2c \ (\exists\, c \in \mathbb{N})$ $\Leftrightarrow$ $a^2 = 4c^2$ $\Leftrightarrow$ a^2 has factor 2	**T**	**F**
(6)	b has factor 2 $\Leftrightarrow$ b^2 has factor 2 $\Leftrightarrow$ $b^2 = 2© \quad (\forall\, © \in \mathbb{N})$	**F** {by (4) and (5)(i)}	
(7)	$b^2 = 2c^2$ $\Leftrightarrow$ $2b^2 = 4c^4$	**F** {special case of (6)(iii)}	
(8)	$2b^2 = a^2$ $\Leftrightarrow$ $\sqrt{2} = \dfrac{a}{b}$ $\Leftrightarrow$ $\sqrt{2} = \dfrac{\alpha}{\beta}$	**F** {by (5)(iii) and (7)(ii)}	**F** {by (5)(iv)}
(9)	$\sqrt{2} \neq \dfrac{\alpha}{\beta}$	**T** {definition of $\neq$}	
(10)	$\forall\, \alpha,\beta \in \mathbb{N}, \sqrt{2} \neq \dfrac{\alpha}{\beta}$ $\stackrel{\text{def}}{\Leftrightarrow}$ $\sqrt{2} \in \mathbb{P}$	**T** {by exact above 9 steps, (9) holds $\forall\, \alpha,\beta \in \mathbb{N}$}	

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