LECTURE ONE

RELIABILITY

Definitions of reliability are as follows;

- Reliability is defined as the 'probability that a specified item will perform a specified function within a defined environment, for a specified length of time.
- It is the probability of an item to perform a required function under specified conditions for a stated period of time.
- It is the probability that a componenet or a system will function without failure for a prescribed period of time under specified condition.

It is worthy of note that other literature sources may define reliability by slightly different statements, regardless of the approach, the three operative phrases "**PERFORM A REQUIRED FUNCTION**", "**UNDER STATED CONDITION**", "**FOR STATED PERIOD OF TIME**", are always emphasized.

Probability is recognizd as a technical word implying "a measure of chance". Mathematically, reliability lies between 0 and 1

Reliability = 1.00 means the item will always work as intended (absolute certainity).
Reliability = 0.90 means 90 per cent likely to work as intended.
Reliability = 0.00 means absolutely certain it will not work as intended (Absolute impossibility)

Figure 1: The Probability Scale

1.1 The Importance of Reliability

Unreliability has a number of unfortunate consequences and therefore for many products and services is a serious threat. Attainment of high reliability of items is important to both the manufacturers and consumers because poor reliable items can have implications on:

- Safety

EXAMPLE

Continuous tests were conducted on an electrical item and faults which were repaired immediately occurred at the following times.

Failures	0	1	1	1	1	1	1
Time (x100 hrs)	$t_0 = 0$	$t_1 = 2$	$t_2 = 4$	$t_3 = 8$	$t_4 = 10$	$t_5 = 14$	$t_6 = 15$

Calculate the mean time between failures (MTBF)

SOLUTION

$$\mathrm{MTBF, m} = \frac{(t_n - t_0)}{n} = \frac{(15 - 0)}{6} \, \mathrm{x}10^2 \mathrm{hrs} = 250\mathrm{hrs}$$

Non-Replacement Method: Non replacement method requires that a large number of the items be put under test and observation made for possible failure at the beginning and the end of the test period. Under this method it is assumed that the test time should be truncated (cut off) before the items are subjected to failure possibly due to wear-out. In other words, the test is limited to the useful life period. Epstein demonstrated that the best estimate of MTBF, m, for a truncated test is given by:

$$m = \frac{\text{test hours for failures} + \text{test hours for survivors}}{\text{number of failures}} \qquad (1.7.1.4)$$

That is

$$m = \frac{\text{total component test hours (survival hours)}}{\text{number of failures}} \qquad (1.7.1.5)$$

EXAMPLE

In order to determine the MTBF of a certain component, 50 were tested for a period lasting 200 hours. The times to failure of the components are shown in table below. 35 components survived without failure. Assuming the wear-out failure can be ignored, calculate:

1. Total test hours before failure
2. Total test hours without failure
3. Total survival hours
4. MTBF

$$M_P3 = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} - \frac{1}{(\lambda_1 + \lambda_2)} - \frac{1}{(\lambda_2 + \lambda_3)} - \frac{1}{(\lambda_1 + \lambda_3)} - \frac{1}{(\lambda_1 + \lambda_2 + \lambda_3)}$$

Where, λ_1, λ_2, λ_3 are the unit failure rates respectively. Furthermore, for an n-unit system each unit having the same failure rate λ, the MTBF is

$$M_{Pn} = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} + \ldots\ldots + \frac{1}{n\lambda}$$

1.7.1.3 MTBF of a Series-Parallel System

This can be obtained by integrating the reliability expression for the series-parallel system. As shown in equations below.

$$M_{sp} = \int_0^\infty R_{sp}(t)dt$$

$$= \int_0^\infty (R_1 R_2 \ldots R_\infty)\,[1 - (1 - Ra)(1 - Rb) \ldots\ldots (1 - Rz)]dt$$

1.7.2 Mean Time to Failure (MTTF)

This term applies to non-repairable items such as capacitors, electric bulbs, resistors, transistors or systems comprised of many parts (spacecraft, microprocessors) etc. It is the average time an item may be expected to function before failure or the average time that elapses until a failure occurs. MTTF can be computed after testing a number of items, N in a specified way (e.g by applying certain electrical, mechanical, heat, or humidity conditions) until all have failed. If the times to failure are (t_1, t_2, t_3,…….., t_n) then the observed MTTF is given by

$$MTTF = \frac{\sum_{i=1}^{N}(t_i - t_0)}{N} \qquad (1.7.2.1)$$

$$MTTF = \frac{(t_1 - t_0) + (t_2 - t_0) + \cdots + (t_N - t_0)}{N} \qquad (1.7.2.2)$$

where

t_0 = starting (reference) time

$(t_1 - t_0)$ = period to 1st failure

$(t_2 - t_0)$ = period to 2nd failure

$(t_N - t_0)$ = period to Nth failure

N = total number of failed components

EXAMPLE

Life testing is made on six (non-repairable) electrical lamps and the following results were obtained. Calculate the MTTF

Failures	0	1	1	1	1	2
Time (x100 hrs)	$t_0 = 0$	$t_1 = 4$	$t_2 = 10$	$t_3 = 16$	$t_4 = 20$	$t_5 = 23$

SOLUTION

$$\mathrm{MTTF} = \frac{(t_1 - t_0) + (t_2 - t_0) + (t_3 - t_0) + (t_4 - t_0) + (t_5 - t_0)}{(1 + 1 + 1 + 1 + 2)})$$

$$\mathrm{MTTF} = \frac{96}{6}\, 10^2$$

$$= 1600 \text{ hours}$$

1.8 Derivation of MTBF, MTTF and failure rate

Derivation of MTBF, MTTF and failure rate from the first principle i.e $m = \frac{1}{\lambda}$. The general expression for MTBF, m, is

$$m = \int_0^\infty R(t)dt \qquad (1.8.1)$$

for the case when λ is constant, $R = e^{-\lambda t}$ and equation (1.8.1) becomes

$$m = \int_0^\infty e^{-\lambda t}\, dt$$

$$= -\left[\frac{1}{\lambda} e^{-\lambda t}\right]_0^\infty$$

$$= \frac{1}{\lambda}[e^{-\infty} - e^{-0}] = \frac{1}{\lambda}$$

$$\mathrm{MTBF}, m = \frac{1}{\lambda} \qquad (1.8.2)$$

If failures are due to chance and if the failure rate is constant, then it follows that

$$\lambda = \frac{1}{\mathrm{MTTF}} \quad \text{for non-repairable items}$$

or

$$\lambda = \frac{1}{\mathrm{MTBF}} \quad \text{for repairable items}$$

Therefore, MTBF can be expressed as the integral of reliability, with the limits of integration from 0 to ∞. i.e.

$$m = \int_0^\infty R(t)dt$$

LECTURE THREE

AVAILABILITY

3.0 AVAILABILITY

Availability is the probability that a system or component will perform its required function at a given instant in time or over a stated period of time when operated and maintained in a prescribed manner. Availability is affected by the failure rate and by maintenance time. Availability formular is given as;

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \quad (3.1$$

The equation of (3.1) shows that availability improvement can be achieved by improving either MTBF or MTTR.

3.1 Analysis of System Availablity

Three form of systems availability shall be discussed namely; steady state availability, instantaneous availability, and mission availability.

3.1.1 Steady- State Availability (A_{ss})

Steady state availability is the proportion of time that a system is available for use when the overall period is of considerable duration. Since availability of a repairable system is a function of its failure rate, λ, and of its repair rate, μ, A_{ss} becomes;

$$\mathrm{A_{ss}} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \quad (3.2)$$

$$A_{ss} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}} \quad (3.3)$$

$$A_{ss} = \frac{\mu}{\mu + \lambda} \quad (3.4)$$

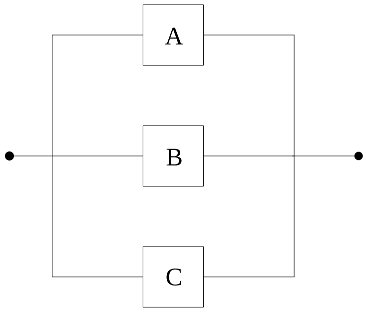
On the other hand, system unavailability can be expressed as;

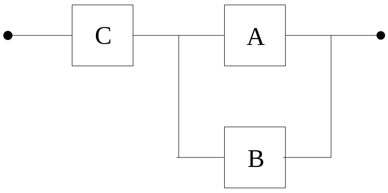
Steady state unavailability , $\overline{\mathrm{A}}_{\mathrm{ss}} = 1 - \mathrm{A_{ss}}$

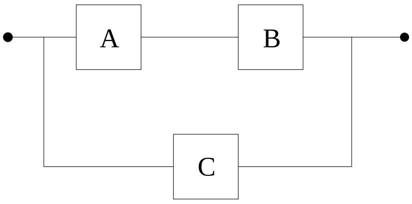
$$\overline{\mathrm{A}}_{\mathrm{ss}} = \frac{\lambda}{\lambda + \mu} \quad (3.5)$$

3.1.2 Intantanneous Availability

Instantaneous availability is the probability that a system will be available at any instant of time, t. Instantaneous availability can be expressed mathematically as;

(ii)


(iii)


(iv)


Recall

$R_A(t) = e^{-\alpha t}$

$R_B(t) = e^{-\beta t}$

$R_C(t) = e^{-\gamma t}$

SOLUTION

Since the system is always in one and only one of the m = 3 finite states, which are mutually exclusive and which together exhaust all possibilities, then the possible no of states the system described above can assume if each block were to be in one of two states are listed below i.e $2^3 = 8$ states

$$
\begin{array}{ccccccccc}
s_0 & = & 0 & 0 & 0 & = & \bar{a} & \bar{b} & \bar{c} \\
s_1 & = & 0 & 0 & 1 & = & \bar{a} & \bar{b} & c \\
s_2 & = & 0 & 1 & 0 & = & \bar{a} & b & \bar{c} \\
s_3 & = & 0 & 1 & 1 & = & \bar{a} & b & c \\
s_4 & = & 1 & 0 & 0 & = & a & \bar{b} & \bar{c} \\
s_5 & = & 1 & 0 & 1 & = & a & \bar{b} & c \\
s_6 & = & 1 & 1 & 0 & = & a & b & \bar{c} \\
s_7 & = & 1 & 1 & 1 & = & a & b & c
\end{array}
$$