Important Results

1. Product of Trigonometric Ratio

(i) $\sin\theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$

(ii) $\cos\theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$

(iii) $\tan\theta \tan(60^\circ - \theta) \tan(60^\circ + \theta) = \tan 3\theta$

(iv) $\cos 36^\circ \cos 72^\circ = \frac{1}{4}$

(v) $\cos A \cos 2A \cos 4A \ldots \cos 2^{n-1} A = \frac{1}{2^n \sin A} \sin(2^n A)$

2. Sum of Trigonometric Ratios

(i) $\sin A + \sin(A + B) + \sin(A + 2B) + \ldots + \sin(A + nB)$

$$= \frac{\sin\left\{A + (n-1)\frac{B}{2}\right\} \sin\frac{nB}{2}}{\sin\frac{B}{2}}$$

(ii) $\cos A + \cos(A + B) + \cos(A + 2B) + \ldots + \cos(A + nB)$

$$= \frac{\sin\frac{nB}{2}}{\sin\frac{B}{2}} \cos\left\{A + \frac{(n-1)B}{2}\right\}$$

3. *A, B and C* are Angles of a Triangle

(i) (a) $\sin(B + C) = \sin A$

(b) $\cos(B + C) = -\cos A$

(c) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

(d) $\cos\left(\frac{B+C}{2}\right) = \sin\frac{A}{2}$

(ii) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(iv) $\sin A + \sin B + \sin C = 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$

(v) $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

(vi) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(vii) $\cot B \cot C + \cot C \cot A + \cot A \cot B = 1$

(viii) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2} \cot\frac{B}{2} \cot\frac{C}{2}$

(ix) $\tan\frac{A}{2} \tan\frac{B}{2} + \tan\frac{B}{2} \tan\frac{C}{2} + \tan\frac{C}{2} \tan\frac{A}{2} = 1$

4. Trigonometric Equations

(i) $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$

d many more)

(ii) $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$

(iii) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

(iv) $\tan\theta = \tan\theta \Rightarrow \theta = n\pi + \alpha, n \in I$