1.**Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate X.**
Step-by-step explanation:

Using the formula for calculating accumulated annuity amount

$F = P \times ([1 + I]^N - 1)/I$

Where P is the payment amount. I is equal to the interest (discount) rate and N number of duration

For 40 years,

$X = 100[(1 + i)^{40} + (1 + i)^{36} + \cdot\cdot\cdot + (1 + i)^4]$

$=[100 \times (1+i)^4 \times (1 - (1 + i)^{40}]/1 - (1 + i)^4$

For 20 years,

$Y = A(20) = 100[(1+i)^{20}+(1+i)^{16}+\cdot\cdot\cdot+(1+i)^4]$

Using X = 5Y (5 times the accumulated amount in the account at the ned of 20 years) and using a difference of squares on the left side gives

$1 + (1 + i)^{20} = 5$

so $(1 + i)^{20} = 4$

so $(1 + i)^4 = 4^{0.2} = 1.319508$

Hence $X = [100 \times (1 + i)^4 \times (1 - (1 + i)^{40})] / 1 - (1 + i)^4$

$= [100 \times 1.3195 \times (1 - 4^2)] / 1 - 1.3195$

$X = 6194.84$

128.804 = 10v^5[(1+k) + $(1+k)^2$v + (1+k)3 v^2.....for infinity}

You can turn this into a geometric progression by pulling out

10 * [(1+k)/1.092]...then you're left with 1 + (1+k)/1.092 + (1+k)^2/1.092^2....for infinity.

Since the problem says k < .092.. you know that (1+k)/1.092 is eventually going to converge to 0.

Therefore, you'll have 1/(1-(1+k)/1.092) as your geometric sum.

That geometric sum * (10*v^6*(1+K)) then has to equal your constant, 128.804.

After dividing 128.804 by 10*v^6 you get 21.84.

21.84 = (1+k)*[1/(1-(1+k)/1.092)

Solving for (1+k), you get 1.04 so k = .04 or 4%

6.**To accumulate 8000 at the end of 3n years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next 2n years. The annual effective rate of interest is i. You are given (1 + ir = 2 Determine i**.

We have that

$$
\begin{aligned}
& 8000 = 98(1+i)^{2n}s_{\overline{n}|i} + 196s_{\overline{2n}|i} = 98(1+i)^{2n}\frac{(1+i)^n-1}{i} + 196\frac{(1+i)^{2n}-1}{i} \\
= \ & (98)2^2\frac{2-1}{i} + 196\frac{2^2-1}{i} = \frac{980}{i}
\end{aligned}
$$

So, $i = \frac{980}{8000} = 0.1225 = 12.25\%$.

7.**Olga buys a 5-year increasing annuity for X. Olga will receive 2 at the end of the first month, 4 at the end of the second month and for each month thereafter the payment increases by 2. The nominal interest rate is 9% convertible quarterly. Calculate X.**

2. The cashflow is:

Payments	2	4	6	$\cdots$	120
Time (in months)	1	2	3	$\cdots$	60

We have $i^{(4)} = 9\%$. So, $i = 9.3083\%$ and $i^{(12)} = 8.933\%$. We use the formula for the increasing annuities with $n = 60$ and $i = 8.933\%/12 = 0.74444\%$:

$$2\,(Ia)_{\overline{n}|i} = 2\cdot\frac{\ddot{a}_{\overline{n}|i} - n\nu^n}{i} = \frac{2(10.158740)}{0.00744} = 2729.21.$$

Solution.

The present value of a perpetuity-immediate starting with a payment of 200 in the first year and increasing by 50 each year thereafter is

$$\frac{150}{i} + \frac{50}{di},$$

and this equals 46,530. We arrive at the following equation

$$\frac{150}{i} + \frac{50(1+i)}{i^2} = 46530,$$

or (by multiplying by i^2, rearranging the terms, and dividing by 10)

$$4653i^2 - 20i - 5 = 0.$$

This is a quadratic equation that solves to

$$i = \frac{20 \pm \sqrt{400 + 20 \cdot 4653}}{2 \cdot 4653} = \frac{10 \pm \sqrt{100 + 5 \cdot 4653}}{4653} \approx \begin{cases} 0.03500025, \\ -0.03070194. \end{cases}$$

The negative solution is unacceptable, so that $i \approx 3.500025\%$.

20.An investor wishes to accumulate 10,000 at the end of 10 years by making level deposits at the beginning of each year. The deposits earn a 12% C: annual effective rate of interest paid at the end of each year. The interest is immediately reinvested at an annual effective interest rate of 8%. Calculate the level deposit.

Solution.

Let x be the level deposit sought. Then we have

$$0.12x \cdot (Is)_{\overline{10}|8\%} + 10x = 10000,$$

so that

$$x = \frac{10000}{0.12 \cdot (Is)_{\overline{10}|8\%} + 10} = \frac{10000}{0.12 \cdot \frac{\ddot{s}_{\overline{10}|8\%} - 10}{0.08} + 10} =$$

$$= \frac{10000}{1.5 \cdot \left(\ddot{s}_{\overline{10}|8\%} - 10\right) + 10} = \frac{10000}{1.5\ddot{s}_{\overline{10}|8\%} - 5} \approx \frac{10000}{18.468232} \approx 541.47035.$$

21A discount electronics store advertises the following financing arrangement: "We don't offer you confusing interest rates. We'll just divide your total cost by 10 and you can pay us that amount each month for a year." The first payment is due on the date of sale and the remaining eleven payments at monthly intervals thereafter. Calculate the effective annual interest rate the store's customers are paying on their loans.

Solution. Since all amounts are proportional to the purchase price, we can simply assume that the purchase price is 100. The customer effectively borrows 100 and pays off that loan in twelve monthly payments of 10 at the beginning of each monthly period. Let j be the effective monthly