PERMUTATION GROUPS

DEFINITION 1: Let S be a finite set. Any bijective mapping from S onto itself is called a **PERMUTATION** on S.

Let f be a permutation on a set $S=\{a_1,a_2,.....,a_n\}$. Then f is denoted by $f=\begin{pmatrix} a_1 & a_2..............a_n \\ f(a_1) & f(a_2).........f(a_n) \end{pmatrix}$.

NOTE 1: The number of permutations on a finite set S with n elements is n!.

The identity mapping on S is known as the identity permutation.

EXAMPLE 1: Let $S=\{1,2,3\}$. Then the set of all permutations on S is $\left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix},\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix},\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}$.

DEFINITION 2: Let f, g be permutations on $S=\{a_1,a_2,.....,a_n\}$. Then we define the product fg by $fg=\begin{pmatrix} a_1 & a_2 & ....a_n \\ (f\circ g)(a_1) & (f\circ g)(a_2).......(f\circ g)(a_n) \end{pmatrix}$.

NOTE 2: The multiplication of permutations is nothing but composition of two bijective mappings on a set.

REMARK 1: We know that composition of mappings is not, in general, commutative. As a result, multiplication of permutations is not, in general, commutative. For example, on the set $S=\{1,2,3\}$, if $f=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$ and $g=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$, then

$$f.g=\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} \text{ but } g.f=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}. \text{ So } f.g\neq g.f.$$

Note carefully, the for fg, $\begin{array}{l} 1\xrightarrow[g]{}3\xrightarrow[f]{}1 \\ 2\xrightarrow[g]{}2\xrightarrow[f]{}3 \\ 3\xrightarrow[g]{}1\xrightarrow[f]{}2. \end{array}$ but for gf, $\begin{array}{l} 1\xrightarrow[f]{}2\xrightarrow[g]{}2 \\ 2\xrightarrow[f]{}3\xrightarrow[g]{}1 \\ 3\xrightarrow[f]{}1\xrightarrow[g]{}3. \end{array}$