and $b = h_2^{-1}(h_1 a) = (h_2^{-1} h_1) a = h_4 a$, where $h_4 = h_2^{-1} h_1 \in H$ (as H is a subgroup of G).

Now $p \in Ha \Rightarrow p = h_5 a$, where $h_5 \in H$

$\Rightarrow p = h_5 h_3 b = h_6 b \in Hb$ [since H being a subgroup of G, $h_6 = h_5 h_3 \in H$]

So, $Ha \subseteq Hb$. ……… **(1)**

Again, $q \in Hb \Rightarrow q = h_7 b$, where $h_7 \in H$

$\Rightarrow q = h_7 h_4 a = h_8 a \in Ha$ [since H being a subgroup of G, $h_8 = h_7 h_4 \in H$]

So, $Hb \subseteq Ha$. ……… **(2)**

By **(1)** and **(2),** $Ha = Hb$.

Hence the result is proved.

THEOREM 2B: Let H be a subgroup of a group G. Then, any two left cosets of H in G are either identical or disjoint.

PROOF: Try yourself.

THEOREM 3A: Let H be a subgroup of a group G. For a, $b \in$ G, $aH = bH$ if and only if $a^{-1}b \in H$.

PROOF: Let us first assume that $a^{-1}b \in H$. Let $a^{-1}b = h \in H$.

Then $b = ah \in aH$. Also, $b = be \in bH$ ['e' is identity element of G].

So, $aH \cap bH \neq \emptyset$. Hence by **THEOREM 2B**, $aH = bH$. [**TO BE PROVED IN EXAM**]

Conversely let $aH = bH$. Then $\exists$ $h_1, h_2 \in H$ such that $ah_1 = bh_2$. Therefore, $a^{-1}b = h_1 h_2^{-1} \in H$. [Since H is a subgroup of G]

THEOREM 3B: Let H be a subgroup of a group G. For a, $b \in$ G, $Ha = Hb$ if and only if $ba^{-1} \in H$.

PROOF: Try yourself.