AMC 12/AHSME
2005

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20 For each x in $[0,1]$, define

$$f(x) = \begin{cases} 2x, & \text{if } 0 \le x \le \frac{1}{2}; \\ 2-2x, & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

Let $f^{[2]}(x) = f(f(x))$, and $f^{[n+1]}(x) = f^{[n]}(f(x))$ for each integer $n \ge 2$. For how many values of x in $[0,1]$ is $f^{[2005]}(x) = \frac{1}{2}$?

(A) 0 **(B)** 2005 **(C)** 4010 **(D)** 2005^2 **(E)** 2^{2005}

21 How many ordered triples of integers (a,b,c), with $a \ge 2$, $b \ge 1$, and $c \ge 0$, satisfy both $\log_a b = c^{2005}$ and $a+b+c = 2005$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

22 A rectangular box P is inscribed in a sphere of radius r. The surface area of P is 384, and the sum of the lengths of its 12 edges is 112. What is r?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

23 Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \ldots, 2^{25}\}$. What is the probability that $\log_a b$ is an integer?

(A) $\frac{2}{25}$ **(B)** $\frac{31}{300}$ **(C)** $\frac{13}{100}$ **(D)** $\frac{7}{50}$ **(E)** $\frac{1}{2}$

24 Let $P(x) = (x-1)(x-2)(x-3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \cdot R(x)$?

(A) 19 **(B)** 22 **(C)** 24 **(D)** 27 **(E)** 32

25 Let S be the set of all points with coordinates (x,y,z), where x, y, and z are each chosen from the set $\{0,1,2\}$. How many equilateral triangles have all their vertices in S?

(A) 72 **(B)** 76 **(C)** 80 **(D)** 84 **(E)** 88