(A) $\frac{1}{2}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{5}{3}$ **(E)** $\frac{9}{5}$

18 The function f has the property that for each real number x in its domain, $1/x$ is also in its domain and

$$f(x) + f\left(\frac{1}{x}\right) = x.$$

What is the largest set of real numbers that can be in the domain of f? **(A)** $\{x|x \neq 0\}$ **(B)** $\{x|x < 0\}$ **(C)** $\{x|x > 0\}$
(D) $\{x|x \neq -1 \text{ and } x \neq 0 \text{ and } x \neq 1\}$ **(E)** $\{-1, 1\}$

19 Circles with centers $(2, 4)$ and $(14, 9)$ have radii 4 and 9, respectively. The equation of a common external tangent to the circles can be written in the form $y = mx + b$ with $m > 0$. What is b?

[img]http://www.artofproblemsolving.com/Forum/album_pic.php?pic_id=5044[/img]

(A) $\frac{908}{199}$ **(B)** $\frac{909}{119}$ **(C)** $\frac{130}{17}$ **(D)** $\frac{911}{119}$ **(E)** $\frac{912}{119}$

20 A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

(A) $\frac{2}{2187}$ **(B)** $\frac{1}{729}$ **(C)** $\frac{2}{243}$ **(D)** $\frac{1}{81}$ **(E)** $\frac{5}{243}$

21 Let

$$S_1 = \{(x, y) \mid \log_{10}(1 + x^2 + y^2) \leq 1 + \log_{10}(x + y)\}$$

and

$$S_2 = \{(x, y) \mid \log_{10}(2 + x^2 + y^2) \leq 2 + \log_{10}(x + y)\}.$$

What is the ratio of the area of S_2 to the area of S_1?

(A) 98 **(B)** 99 **(C)** 100 **(D)** 101 **(E)** 102

22 A circle of radius r is concentric with and outside a regular hexagon of side length 2. The probability that three entire sides of hexagon are visible from a randomly chosen point on the circle is $1/2$. What is r?

(A) $2\sqrt{2} + 2\sqrt{3}$ **(B)** $3\sqrt{3} + \sqrt{2}$ **(C)** $2\sqrt{6} + \sqrt{3}$ **(D)** $3\sqrt{2} + \sqrt{6}$
(E) $6\sqrt{2} - \sqrt{3}$

23 Given a finite sequence $S = (a_1, a_2, \ldots, a_n)$ of n real numbers, let $A(S)$ be the sequence

$$\left(\frac{a_1 + a_2}{2}, \frac{a_2 + a_3}{2}, \ldots, \frac{a_{n-1} + a_n}{2}\right)$$