Mathematic

(Chapter – 4) (Quadratic Equations)

(Class X)

Exercise 4.1

Question 1:

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$ (iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$ (vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Answer 1:

(i) $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1) \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2 \Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x \Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow x = \pm\frac{\sqrt{33}}{4} - \frac{1}{4}$

$\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$

$\Rightarrow x = \frac{\sqrt{33} - 1}{4}$ or $\frac{-\sqrt{33} - 1}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right) = 0$ and $\left(2x + \sqrt{3}\right) = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

Answer 2:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant $(b^2 - 4ac)$ will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$,

we obtain $a = 2$, $b = k$, $c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = k$, $b = -2k$, $c = 6$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

For equal roots, $b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Comparing this equation with $al^2 + bl + c = 0$,

we obtain $a = 1$, $b = -40$, $c = 400$

Discriminate $= b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m