If $\lim_{n \to \infty} \sqrt[n]{|a_n|} = L$ and $L > 1$ or $\lim_{n \to \infty} \sqrt[n]{|a_n|} = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

If $\lim_{n \to \infty} \sqrt[n]{|a_n|} = L$ and $L = 1$, then there is no conclusion about the convergence or divergence of $\sum_{n=1}^{\infty} a_n$.

The Comparison Test

If series $\sum a_n$ and $\sum b_n$ have positive terms, $\sum b_n$ is convergent, and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

If series $\sum a_n$ and $\sum b_n$ have positive terms, $\sum b_n$ is divergent, and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

The Integral Test

For a function f that is a continuous, positive, and decreasing function on $[1, \infty\rangle$ and $a_n = f(n)$, the following is true:

The series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if $\int_1^{\infty} f(x)\, dx$ is convergent.