3.2 Method of Transformation of Variables.

3.2.1 Homogeneous Equations.

We consider homogeneous first order differential eqns and the methods for solving them.

Def: A function $f(x,y)$ is said to be homogeneous of degree n if on replacing x and y by λx & λy (where λ is a parameter) we have

$$f(\lambda x, \lambda y) = \lambda^n f(x,y). \quad \text{—— (1)}$$

Example 1:

$f(x,y) = x^4 - x^3 y$ is homogeneous of degree 4 since

$$\begin{aligned} f(\lambda x, \lambda y) &= (\lambda x)^4 - (\lambda x)^3 (\lambda y) \\ &= \lambda^4 x^4 - \lambda^4 x^3 y \\ &= \lambda^4 (x^4 - x^3 y) \\ &= \lambda^4 f(x,y) \end{aligned}$$

Example 2:

$f(x,y) = e^{y/x} + \tan(y/x)$ is homogeneous of degree 0, since

$$\begin{aligned} f(\lambda x, \lambda y) &= e^{(\lambda y)/(\lambda x)} + \tan\left(\frac{\lambda y}{\lambda x}\right) \\ &= e^{y/x} + \tan(y/x) \\ &= \lambda^0 \left(e^{y/x} + \tan(y/x)\right) \\ &= \lambda^0 f(x,y). \end{aligned}$$

Example 3:

$f(x,y) = x^2 + \sin x \cos y$ is NOT homogeneous.

$$\begin{aligned} f(\lambda x, \lambda y) &= (\lambda x)^2 + \sin(\lambda x)\cos(\lambda y) \\ &= \lambda^2 x^2 + \sin(\lambda x)\cos(\lambda y) \\ &\neq \lambda^n f(x,y) \quad (\text{for any value of } n). \end{aligned}$$