Ex: 1) In example (3) above, show that $y = x^2$ is NOT a solution to the given DE

2) Find another solution to the DE in e.g. (4) for all $x \in \mathbb{R}$.

General and Particular Solution

Consider the first order DE:

$$\frac{dy}{dx} = 3$$

Integrating with respect to x yields the general solution

$$y = 3x + C, \qquad C = \text{constant}.$$

The general solution of the differential equation, which includes all possible solutions, is a family of straight lines with slope equal to 3. On the other hand, $y = 3x$ is a particular solution passing through the origin, with the constant C being 0.

Consider the differential equation

$$\frac{d^3y}{dx^3} = 48x.$$

Integrating both sides of the equation with respect to x gives

$$\frac{d^2y}{dx^2} = 24x^2 + C_1.$$

Integrating with respect to x again yields

$$\frac{dy}{dx} = 8x^3 + C_1x + C_2.$$

Integrating with respect to x once more results in the general solution

$$y = 2x^4 + \tfrac{1}{2}C_1x^2 + C_2x + C_3,$$

where C_1, C_2, C_3 are arbitrary constants. When the constants C_1, C_2, C_3 take specific values, one obtains particular solutions. For example,

$$y = 2x^4 + 3x^2 + 1, \qquad C_1 = 6, \; C_2 = 0, \; C_3 = 1,$$
$$y = 2x^4 + x^2 + 3x + 5, \qquad C_1 = 2, \; C_2 = 3, \; C_3 = 5,$$

are two particular solutions.

Remarks: In general, an nth-order ordinary differential equation will contain n arbitrary constants in its general solution. Hence, for an nth-order ordinary differential equation, n conditions are required to determine the n constants to yield a particular solution.

In applications, there are usually two types of conditions that can be used to determine the constants.