(27) Let $\overline{X}$ denote the mean of a random sample of size 100 from the distribution that has probability density function

$$f(x) = 9xe^{-3x} \quad x > 0$$

Determine the sampling distribution of $\overline{X}$ and hence compute the probability that $0.612 < \overline{X} < 0.708$.

(28) Let $\overline{X}$ be the sample mean of a random sample of size 81 drawn from population that has probability density function

$$f(y) = \begin{cases} \frac{1}{4}xe^{-\frac{1}{8}x^2}, & 0 < x < \infty \\ 0, \text{elsewhere} \end{cases}$$

Find $\Pr(1.84 < \overline{X} < 3.02)$.

(29) Let $\overline{X}$ be the mean of a random sample of size 121 from the distribution that has probability density function

$$f(x) = 4(1-x)^3, \quad 0 < x < 1$$

Use the large sampling distribution to compute $\Pr(0.4 < \overline{X} < 0.6)$

(30)]Let X_1, X_2, X_3 and X_4 have joint probability density function

$$f(x_1, x_2, x_3, x_4) = \begin{cases} e^{-(x_1+x_2+x_3+x_4)}, & x_i > 0, i = 1, 2, 3, 4 \\ 0 \text{ elsewhere} \end{cases}$$

Find the probability density function of $Z = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$.

(31) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from the distribution

$$f(x) = 1 \quad 0 < x < 1$$

Further let $Y = max(X_1, X_2, ..., X_n)$. Using the distribution function technique, find the probability density function of Y. Hence find its mean and variance values.

(32) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from the distribution

$$f(x) = 5(1-x)^4 \quad 0 < x < 1$$

Further let $Z = min(X_1, X_2, ..., X_n)$.
Find the probability density function of Z and hence compute the mean of Z

(33) The joint probability density of X_1, X_2, X_3 is

$$f(x_1, x_2, x_3) = \begin{cases} e^{-(x_1+x_2+x_3)} & x_i > 0 \quad i = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$