In this case the usual differentiations and algebra yield

$$x_p = t + 1,$$

and so

$$x = x_c + x_p = c_3 \cos t + c_4 \sin t + t + 1. \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (15)$$

Now c_3 and c_4 can be expressed in terms of c_1 and c_2 by substituting (14) and (15) into either equation of (12). By using the second equation, we find, after combining terms,

$$(-c_1 - c_4)\sin t + (c_2 - c_3)\cos t = 0,$$

so $-c_1 - c_4 = 0$ and $c_2 - c_3 = 0$. Solving for c_3 and c_4 in terms of c_1 and c_2 gives

$$c_3 = c_2 \qquad \text{and} \qquad c_4 = -c_1.$$

Finally, a solution of (12) is found to be

$$x(t) = c_2 \cos t - c_1 \sin t + t + 1,$$
$$y(t) = c_1 \cos t + c_2 \sin t + t - 1.$$

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(22) Solve the given problem

$$\frac{dx}{dt} = y - 1$$
$$\frac{dy}{dt} = -3x + 2y \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (16)$$

Solution:

First we write the system in differential operator notation:

$$Dx - y = -1$$
$$(D - 2)y + 3x = 0. \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (17)$$

Then, by eliminating x, we obtain

$$D^2x - 2Dx + 3x = 2$$

or

$$(D^2 - 2D + 3)x = 2.$$

Since the roots of the **auxiliary equation** $m^2 - 2m + 3 = 0$ are $m_1 = 1 - \sqrt{2}i$ and $m_2 = 1 + \sqrt{2}i$, the complementary function is

$$x_c = e^t\left(c_1 \cos \sqrt{2}t + c_2 \sin \sqrt{2}t\right).$$

To determine the particular solution x_p, we use undetermined coefficients by assuming that

$$x_p = A.$$

Therefore

$$x_p' = 0, \qquad x_p'' = 0,$$

so,

$$x_p'' - 2x_p' + 3x_p = 0 - 0 + 3A = 2.$$

The last equality implies that

$$A = \frac{2}{3};$$

Thus,

$$x = x_c + x_p = e^t\left(c_1 \cos \sqrt{2}t + c_2 \sin \sqrt{2}t\right) + \frac{2}{3}. \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (18)$$